



ACTUAL PROBLEMS OF MODERN SCIENCE, EDUCATION AND TRAINING

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Section 1. MODERN PROBLEMS OF PEDAGOGY AND PSYCHOLOGY

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THE METHODOLOGICAL ESSENCE OF THE AXIOLOGICAL APPROACH AND THE MECHANISM OF INTERIORIZING ECOLOGICAL VALUES

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Annotatsiya. Ushbu maqolada pedagogik tadqiqotlarda aksiologik yondashuvning metodologik mohiyati hamda shaxs tomonidan ekologik qadriyatlarni interiorizatsiya qilish mexanizmi chuqur tahlil qilinadi. Tadqiqot maqsadi — qadriyat nazariyasining falsafiy va psixologik asoslarini, jumladan, qadriyatni subyekt-obyekt munosabati sifatida qarashni umumlashtirish hamda ekologik qadriyatlarning tashqi ijtimoiy-madaniy shakldan ichki shaxsiy mazmunga aylanish bosqichlarini aniqlashdan iborat. Tadqiqotda tizimli tahlil, tarixiy-mantiqiy tahlil, qiyosiy-nazariy tahlil va kontent-tahlil usullari qo‘llanildi. Natijada qadriyatlarning ierarxik tuzilishi va ularning shaxs ongida bajaradigan vazifalari oydinlashtirildi, shuningdek, ekologik qadriyatlarni interiorizatsiya qilishning to‘rt bosqichli — kognitiv-axborot, hissiy-baholovchi, faoliyat-amaliy va shaxsiy-refleksiv — nazariy modeli hamda har bir bosqichning ichki mexanizmi ishlab chiqildi. Olingan xulosalar ta’lim muassasalarida ekologik ta’lim jarayonini ilmiy asoslangan holda loyihalashda amaliy ahamiyatga ega.

Kalit so‘zlar: aksiologik yondashuv, qadriyat, qadriyatlar ierarxiyasi, ekologik qadriyatlar, interiorizatsiya, shaxsiy ma’no, ekologik ta’lim.

Аннотация. В данной статье представлен углублённый анализ методологической сущности аксиологического подхода в педагогических исследованиях, а также механизма интериоризации экологических ценностей личностью. Цель исследования — обобщить философские и психологические основы теории ценностей, включая понимание ценности как субъектно-объектного отношения, и выявить этапы перехода экологических ценностей из внешней социокультурной формы во внутреннее личностное содержание. В работе использованы методы системного анализа, историко-логического анализа, сравнительно-теоретического анализа и контент-анализа. В

результате были уточнены иерархическая структура ценностей и их функции в сознании личности, а также разработана четырёхэтапная теоретическая модель интериоризации экологических ценностей (когнитивно-информационный, эмоционально-оценочный, деятельностно-практический и личностно-рефлексивный этапы) с описанием внутреннего механизма каждого этапа. Полученные результаты имеют практическое значение для научно обоснованного проектирования процесса экологического образования в образовательных учреждениях.

Ключевые слова: *аксиологический подход, ценность, иерархия ценностей, экологические ценности, интериоризация, личностный смысл, экологическое образование.*

Abstract. This article provides an in-depth analysis of the methodological essence of the axiological approach in pedagogical research, together with the mechanism through which an individual interiorizes ecological values. The purpose of the study is to synthesize the philosophical and psychological foundations of value theory — including its treatment of value as a subject-object relation — and to identify the stages through which ecological values move from an external socio-cultural form into internal personal content. The study employed methods of systemic analysis, historical-logical analysis, comparative-theoretical analysis, and content analysis. As a result, the hierarchical structure of values and their functions in personal consciousness were clarified, and a four-stage theoretical model of the interiorization of ecological values — cognitive-informational, emotional-evaluative, activity-practical, and personal-reflective — was developed together with the inner mechanism of each stage. The findings have practical significance for the scientifically grounded design of the ecological education process in educational institutions.

Key words: *axiological approach, value, hierarchy of values, ecological values, interiorization, personal meaning, ecological education.*

Introduction

One of the significant features of the development of modern society is the ever-deepening contradiction in the relationship between man and nature. This contradiction has not only ecological but also profound value-based and spiritual roots, and one way of resolving it is regarded as forming in the individual a stable, internally felt value-based attitude toward nature. Effectively addressing this task requires pedagogical science to deeply understand the methodological potential of the axiological approach, as well as to scientifically substantiate the mechanism through which a value moves from being an external social requirement or norm into an individual's internal conviction — that is, the mechanism of interiorization.

The “Concept for the Development of Environmental Education in the Republic of Uzbekistan” [6], approved by Resolution No. 434 of the Cabinet of Ministers of the Republic of Uzbekistan dated May 27, 2019, identifies raising the ecological awareness and culture of the population, particularly of the younger generation, as a priority task. In practice, however, environmental education is often limited to the level of transmitting information: a child or student acquires knowledge about the need to protect nature, but this knowledge does not become part of their personal value system and, consequently, does not affect real behavior. This situation demonstrates the need to understand the mechanism of value formation not superficially, but from a deep psychological-pedagogical perspective.

The purpose of the article is to elucidate the methodological essence of the axiological approach, beginning from its philosophical and psychological roots, and to theoretically substantiate the stages of the mechanism of interiorizing ecological values, their internal structure, and their interrelationship.

Literature Review

Value theory — axiology — took shape as an independent field of philosophy at the end of the 19th and the beginning of the 20th centuries, although its pedagogical-psychological interpretation became active only in the second half of the 20th century. V.P. Tugarinov, in his work devoted to value theory [1, pp. 21–24], defines value as a property of an object to satisfy a subject's needs, that is, as the outcome of a subject-

object relation, and divides societal values into material, socio-political, and spiritual types. M.S. Kagan [8, pp. 45–47], in his philosophical theory of value, divides values into vital-utilitarian, socio-political, religious, and artistic-aesthetic types, singling out ecological values as a distinct group of values that regulates the relationship between man and nature. M. Rokeach, in *The Nature of Human Values* [2, pp. 12–14], divides values into terminal (end-state) and instrumental (means) values, demonstrating their function in guiding individual behavior and shaping choice — a classification that also makes it possible to analyze ecological values both as a terminal value, such as “harmony with nature,” and as an instrumental value, such as “resource conservation.”

The psychological foundations of the mechanism of interiorization are elaborated in depth in L.S. Vygotsky's cultural-historical theory [3, pp. 56–57]: according to this theory, higher mental functions initially arise in a social, interpersonal (interpsychic) form and only later become an individual's internal, personal (intrapsychic) content. S.L. Rubinstein's [9, pp. 118–120] well-known methodological principle that “external causes act through internal conditions” indicates that an external value-based influence (education, upbringing, social environment) becomes a genuine internal value only as a result of being “refracted” through an individual's personal characteristics, experience, and emotional state. P.Ya. Galperin [4, pp. 33–35], in his theory of the stage-by-stage formation of mental actions, describes in detail the mechanism by which an external, material action is transformed into an internal, mental image, showing that this process consists of several distinct stages — an approach that also carries methodological significance for analyzing the interiorization of values. D.A. Leontiev's [11, pp. 154–156] theory of personal meaning places particular emphasis on the transition of a value in an individual's consciousness from being “objectively significant” to being “mine,” “important to me” — and it is precisely this transition that constitutes the final outcome of interiorization. A.N. Leontiev's activity theory [5, pp. 89–91], for its part, shows that values are intrinsically linked to the motives of an individual's activity, and that a value becomes a stable personal structure only once it has been “tested” in activity.

Regarding the educational significance of ecological values, K.D. Ushinsky's [10, pp. 15–17] classical ideas on the role of nature in cultivating children's feelings deserve

particular attention: in his view, direct contact with nature shapes in a child not only knowledge but also a profound emotional-aesthetic and moral attitude. In international practice, the professional standards developed by the National Association for the Education of Young Children (NAEYC) of the United States [12, pp. 22–24] define the practical mechanisms for shaping value-oriented educational content in the work of early childhood educators.

The theories reviewed above provide a sufficiently rich methodological foundation for describing the mechanism of interiorizing ecological values. However, most of these theories are either of a general psychological nature, not linked to a specific ecological context, or, conversely, focus solely on the substantive side of environmental education without illuminating the stage-by-stage mechanism of value formation. It is precisely this gap — the need to unite various theoretical approaches into a single, staged model adapted to ecological values — that defines the scientific novelty of the present study.

Research Methodology

The study is theoretical-methodological in nature and employed the following methods: 1) systematic analysis of the philosophical, psychological, and pedagogical literature on the topic; 2) comparative-theoretical analysis of various axiological and interiorization theories in order to identify their common and distinctive features; 3) historical-logical analysis to trace the stages in the development of value theory; 4) theoretical modeling of a staged model of the process of interiorizing ecological values, based on the theoretical conclusions obtained; and 5) an assessment of the directions in which the developed model may be applied to pedagogical practice. The object of the study was the methodological function of the axiological approach within the pedagogical process, and its subject was the mechanism of interiorization of ecological values and its staged structure.

Results and Discussion

The theoretical analysis conducted showed that the methodological essence of the axiological approach manifests itself in several interrelated aspects. First, it treats value as a unity of the objective (social significance recognized by society) and the subjective (personal meaning perceived by the individual), which makes it necessary to design

educational content not only from the standpoint of “correct” knowledge but also from that of personally accepted values. Second, it makes it possible to analyze the value system hierarchically — at the universal-human, national, socio-group, and personal levels — with ecological values emerging as a group of values that unites all of these levels and possesses a “cross-cutting” character. Third, the axiological approach serves as the methodological factor guiding the transition “from knowledge to value” in the design of educational content, meaning that the ultimate goal of the pedagogical process is defined not merely as the transmission of information but as the formation of an individual's internal value-based attitude.

Based on a synthesis of the theories analyzed above, a four-stage theoretical model of the interiorization of ecological values was developed, with each stage possessing its own distinct internal mechanism:

1) The cognitive-informational stage — the individual acquires knowledge about nature, ecological problems, and their cause-and-effect relationships; at this stage the knowledge still has an “external” character and, in Rubinstein's terms, has not yet been “refracted” through internal conditions, meaning it has not yet acquired personal significance;

2) The emotional-evaluative stage — the acquired knowledge is processed through the individual's feelings, empathy toward nature, and aesthetic perception; the emotional-aesthetic attitude emphasized by Ushinsky begins to take shape precisely at this stage, resulting in a positive or negative personal attitude toward the knowledge;

3) The activity-practical stage — in accordance with Galperin's theory, the individual “tests” the acquired knowledge and attitude through practical activity (such as protecting nature, sorting waste, and conserving resources); as A.N. Leontiev noted, it is precisely at this stage that the value begins to become a motive for activity, meaning that the individual begins to act not because of external demand but because of internal need;

4) The personal-reflective stage — the individual consciously evaluates and analyzes his or her own behavior from the standpoint of ecological values; according to D.A. Leontiev's theory, it is precisely at this stage that the value is transformed from

being “objectively significant” into a stable personal meaning intrinsically connected with the individual's “self.”

These four stages are not linear but spiral in character: when an individual encounters new ecological situations, the process may begin anew, although each time it may proceed at a higher level — one that is more conscious and more stable. For this reason, limiting the design of the environmental education process to the cognitive-informational stage alone (the provision of information) is, from the standpoint of the axiological approach, methodologically insufficient: educational institutions must consistently organize means of emotional influence (works of art, direct contact with nature), forms of practical activity (projects, ecological practice), and special pedagogical conditions for reflection (discussion, self-assessment).

Conclusion and Recommendations

The theoretical study conducted showed that the axiological approach constitutes an effective and multifaceted methodological foundation for understanding the process of value formation in pedagogy, and that the interiorization of ecological values manifests itself not as a linear process but as a complex, spiral process consisting of four interrelated stages (cognitive-informational, emotional-evaluative, activity-practical, and personal-reflective). On this basis, the following recommendations were developed:

- to enrich environmental education programs, beyond merely informational content, with means of emotional-aesthetic influence (fiction, direct contact with nature);
- to expand opportunities for “testing” values in activity through ecological practice in educational institutions (nature-conservation projects, ecological campaigns, resource-saving initiatives);
- to introduce regular elements of reflection into the educational process (discussion, essay writing, self-assessment of one's own behavior), since it is precisely through reflection that a value becomes a stable internal conviction of the individual;
- to introduce specialized professional-development modules that train educators and teachers in methods of environmental education based on the axiological approach;
- to develop diagnostic tools aimed at determining the level (stage) of interiorization of ecological values among students and learners.

The implementation of these recommendations will contribute to elevating the process of ecological education to a qualitatively new, personality-oriented stage, and to shaping in the rising generation an ecological consciousness based on a stable, internal system of values.

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TEACHING READING LITERACY TO FUTURE PRIMARY SCHOOL TEACHERS: CONTENT, STRUCTURAL COMPOSITION AND STAGES OF PREPARATION

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Annotatsiya. Maqolada bo'lajak boshlang'ich sinf o'qituvchilarini o'qish savodxonligini o'rgatishga tayyorlash metodikasining mazmuni, tarkibiy tuzilishi va bosqichlari nazariy jihatdan tahlil qilinadi. Xalqaro adabiyotlarga tayangan holda o'qitish bo'yicha bilim komponentlari – til tuzilishi bilimi, o'qitish metodlari, tushunishni rivojlantirish va baholash – ajratiladi hamda ularni oliy ta'limda shakllantirishning to'rt bosqichli modeli taklif etiladi. Tadqiqot nazariy-tahliliy xarakterga ega bo'lib, natijalar keyingi empirik tekshiruvlar uchun asos bo'lishi mumkin.

Kalit so'zlar: o'qish savodxonligi, bo'lajak boshlang'ich sinf o'qituvchisi, o'qitish metodikasi, metodik tayyorgarlik, o'qitish bilimi, oliy pedagogik ta'lim.

Аннотация. В статье теоретически анализируются содержание, структура и этапы подготовки будущих учителей начальных классов к обучению чтению. На основе зарубежных исследований выделяются компоненты знаний об обучении чтению (знание языковой структуры, методы обучения, развитие понимания текста и оценивание) и предлагается четырёхэтапная модель их формирования в вузе.

Ключевые слова: грамотность чтения, будущий учитель начальных классов, методика обучения, методическая подготовка, знания об обучении чтению.

Abstract. The article provides a theoretical analysis of the content, structure and preparation stages of future primary school teachers for teaching reading literacy. Drawing on international research, four components of knowledge for teaching reading are identified (knowledge of language structure, teaching methods, development of comprehension, and assessment), and a four-stage model of their formation in university study is proposed.

Key words: *reading literacy, future primary school teacher, teaching methodology, methodological preparation, knowledge for teaching reading.*

Introduction

Reading literacy is one of the core outcomes of primary education. In the primary grades pupils acquire the basic skills of decoding, fluent reading and text comprehension, and these skills determine their progress in almost every later subject. Reforms in Uzbekistan have raised expectations for teacher preparation, yet the specific content of knowledge for teaching reading is rarely defined in operational terms.

International evidence points in the same direction. A study in Kazakhstan found that pre-service primary teachers had limited ability to teach reading literacy and to interpret international assessment results [1]. A Spanish study reported that future teachers had little reading experience of their own and judged their methodological preparation at university as insufficient [2]. A study in the United States found that pre-service teachers' knowledge of reading instruction differed according to the programme they attended and the number of reading methods courses they had completed [3].

These findings raise three research questions. RQ1: What components make up the knowledge that a future primary teacher needs for teaching reading literacy? RQ2: How can these components be developed during university study? RQ3: In what sequence should this preparation be organised across the years of the bachelor's programme? The aim of the article is to answer these questions on a theoretical basis and to propose a structured model of preparation.

Literature Review

Research on reading acquisition distinguishes between word recognition and language comprehension. Ehri's theory of orthographic mapping describes how readers connect the spellings, pronunciations and meanings of words, and how these connections are stored for fast recognition [4]. Teachers therefore need explicit knowledge of phonemes and letter-sound relationships, since instruction that ignores them can leave pupils dependent on guessing.

The report prepared by the National Research Council in the United States identified the elements of reading instruction that are important for preventing reading difficulties in young children, including phonological awareness, phonics, fluency, vocabulary, comprehension and the assessment of reading progress [5].

Scarborough's work on early language and literacy shows that weaknesses in early language skills and phonological processing are linked to later reading difficulties [6]. This connection makes early reading instruction a priority in primary education.

Studies of teacher knowledge show that pre-service teachers' understanding of reading instruction is uneven. Clark, Helfrich and Hatch found that elementary pre-service teachers had fairly strong knowledge of phonological awareness, phonics, comprehension and vocabulary, but that their knowledge differed significantly between two programmes and according to the number of methods courses completed [3]. Álvarez-Álvarez and Diego-Mantecón showed that future primary teachers describe their own reading histories as limited and their university training as inadequate, although they expressed a clear commitment to promoting reading once they become teachers [2]. Taken together, these studies suggest that knowledge alone is not enough; the way training is organised matters.

In Uzbekistan, the issue has been discussed in relation to the special professional competence of primary teachers. Eshonkulova argues that future primary teachers need a special pedagogical competence and links this need to the weak reading and text-comprehension results of Uzbek pupils in PIRLS 2021 [7]. Boltayeva describes reading literacy as a foundation for pupils' general, personal and social development, identifies decoding, vocabulary and inference as its core components, and discusses interactive, differentiated and technology-based approaches for developing them [8]. However, they do not offer a structured sequence for developing these components during university study.

Fuller's concerns theory describes the development of teachers as a movement from concerns about the self, to concerns about tasks, and finally to concerns about pupils' learning [9]. The theory suggests that early training should focus on the student's own understanding and confidence, while later training should focus on the learning

outcomes of pupils. International assessment data provide external evidence of the same need: the PIRLS 2021 international report shows that pupils' reading achievement differs considerably across countries [10].

The literature describes the knowledge components clearly, but their step-by-step formation during a four-year bachelor's programme has received less attention. This gap defines the contribution of the present study.

Research Methodology

The study uses a theoretical and analytical design. Its purpose is to synthesise existing knowledge into a structured model rather than to test a hypothesis with new empirical data.

Sources were selected according to three criteria: (a) relevance to reading instruction or to the preparation of primary teachers; (b) publication in peer-reviewed journals, academic books or national reports; and (c) availability in English, Russian or Uzbek. Priority was given to studies that examined pre-service teachers directly. The final synthesis includes ten sources.

The analysis proceeded in three stages. First, each source was read and the statements about teacher knowledge, training practices and outcomes were extracted. Second, the extracted statements were grouped into thematic categories, which formed the four components of the model. Third, the components were arranged in a sequence according to the complexity of the knowledge involved and the typical structure of a bachelor's curriculum in primary education. This sequence became the four-stage model.

The grouping is interpretive; to reduce subjectivity, each component was checked against at least two independent sources. The method cannot establish the effectiveness of the model, which requires empirical testing.

Results and Discussion

The synthesis yields four components of knowledge for teaching reading literacy: knowledge of language structure, knowledge of teaching methods, knowledge for developing comprehension, and knowledge for assessment. Table 1 summarises the

content of each component, the typical ways in which it is developed in training, and an observable indicator of its formation.

Table 1. Components of future primary teachers' knowledge for teaching reading

Component	Content	Development method	Indicator
Language structure knowledge	Phonological awareness, letter–sound correspondence, word structure	Language theory course, analytical tasks	Segmenting phonemes and linking them to letters
Teaching methods	Methods of teaching decoding and fluent reading	Methodology courses, micro-teaching	Justifying a method by pupils' age and level
Developing comprehension	Vocabulary, text structure, comprehension strategies	Text-based lesson design, discussion	Designing questions and tasks for a text
Assessment	Diagnostics, interpretation of results, international assessment data	Practice-based observation, analytical reports	Analysing a pupil's results and giving recommendations

Language structure knowledge is the foundation of the model. Without a clear understanding of phonemes, letter–sound relationships and word structure, the teacher cannot identify the source of a pupil's difficulty. This component corresponds to the mechanisms of word learning described by Ehri [4] and to the foundational skills emphasised in the National Research Council report [5].

Teaching methods knowledge concerns the selection and justification of approaches to decoding and fluent reading. The Kazakh and Uzbek studies suggest that teachers often know methods in theory but cannot apply them to particular pupils [1, 7]. For this reason, methods knowledge should be developed not only through lectures but also through the analysis of lesson plans and micro-teaching with feedback.

Comprehension knowledge covers vocabulary development, the structure of texts and strategies for inference and interpretation. Boltayeva's identification of vocabulary

and inference as core components supports this distinction [8]. Pupils who decode well may still fail to understand what they read, so this component cannot be reduced to technical reading skill.

Assessment knowledge links the other three components. The Kazakh study showed that the largest gap between theory and practice appeared in interpreting international assessment data [1]. Assessment knowledge therefore includes the ability to select or design diagnostic tasks, to interpret the results and to recommend changes to instruction. Its inclusion reflects the importance of large-scale assessment [10].

The four components are not developed at the same time. The stage model in Table 2 arranges them in a sequence that matches the progression of the bachelor's curriculum: foundational knowledge of language structure in the first year, methods in the second, integration of comprehension in practical courses in the third, and diagnosis and independent assessment in the fourth. The sequence follows Fuller's movement from concerns about the self to concerns about pupils' learning [9].

Table 2. Stages of preparation for teaching reading during university study

Stage (course)	Leading task	Dominant component	Expected result
1. Introduction (Year 1)	Becoming familiar with the reading process and key concepts of reading literacy	Language structure knowledge	Describing the basic concepts
2. Methodological basis (Year 2)	Mastering methods for teaching decoding and fluent reading	Teaching methods	Selecting and justifying methods
3. Practical integration (Year 3)	Applying methods in micro-teaching and short-term practice	Developing comprehension	Designing and delivering a reading lesson
4. Assessment and independence (Year 4)	Diagnosing and analysing pupils' reading results	Assessment	Independent diagnostics and recommendations

Each stage can be described by three mastery levels: reproducing a model procedure, applying knowledge independently in typical situations, and adapting it to

new pupils with justified choices. These levels require specific assessment instruments in future research.

The model implies that courses should be linked to one another. The methods course in the second year should build on the language structure knowledge acquired in the first year, and the practice in the third year should draw on the methods learned in the second. Such alignment matters more than the number of courses alone [3].

Conclusion and Recommendations

The article has shown that teachers' knowledge for teaching reading literacy consists of four interrelated components formed through a four-stage sequence during university study: language structure, teaching methods, comprehension and assessment (RQ1); developed through specific course and practice activities (RQ2) and following the years of the bachelor's programme (RQ3).

On the basis of the analysis, the following recommendations are made:

- 1) courses of the primary education programme should be arranged so that each stage builds on the knowledge developed in the previous one;
- 2) pedagogical practice should begin earlier and include the analysis of real pupils' reading results;
- 3) assessment competence should be developed through diagnostic projects in which students analyse national and international assessment data;
- 4) the proposed components and levels should be used to monitor students' progress, and the model should be tested empirically with students in Uzbekistan.

A limitation is that the model has not yet been tested empirically; future research should examine its validity with student samples.

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Section 2. MODERN PROBLEMS OF TECHNICAL SCIENCES

UDC: 666.65:620.17

MECHANICAL PROPERTIES AND STRENGTHENING TECHNOLOGY FOR ADVANCED CERAMIC COMPOSITES

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Annotatsiya: Ushbu maqolada ilg'or keramika-kompozit materiallarning mexanik xossalari va ularni mustahkamlash texnologiyasiga qo'yiladigan asosiy talablar bayon etilgan. Yuqori samarali keramika-kompozitlarni ishlab chiqarishda qo'llaniladigan asosiy materiallar qattqlik, zarbiy qovushqoqlik va mustahkamlik kabi muhim mexanik xossalari nuqtai nazaridan tahlil qilinib, turli yetakchi tadqiqotchilarning natijalari o'zaro taqqoslangan. Shuningdek, maqolada faza o'zgarishlari, mikroyoriqlar, dislokatsiyalar, tolalar va metall zarralari kabi turli mustahkamlash mexanizmlarining ta'siri chuqur o'rganilgan.

Kalit so'zlar: keramika, qattqlik, mustahkamlik, zarbiy qovushqoqlik, mustahkamlash, konstruksion keramika.

Аннотация: В данной статье представлены основные требования к механическим свойствам и технологиям упрочнения современных керамических композиционных материалов. Проведен анализ материалов, наиболее часто используемых для изготовления высокоэффективных керамических композитов, с точки зрения таких ключевых механических характеристик, как твердость, ударная вязкость и прочность, на основе сопоставления результатов исследований ведущих специалистов. Кроме того, в работе подробно рассмотрено влияние различных механизмов упрочнения, обусловленных фазовыми превращениями, микротрещинами, дислокациями, а также введением волокон и металлических частиц.

Ключевые слова: керамика, твердость, прочность, ударная вязкость, упрочнение, конструкционная керамика.

Abstract: This article presents the main requirements on mechanical properties and strengthening technology for advanced ceramic composite materials. The basic materials mainly used in fabrication high-performance ceramic composites have been analyzed in terms of critical mechanical properties like hardness, impact toughness and strength comparing the results of various leading researchers. Additionally, the effect of various strengthening mechanisms like phase, microcracks, dislocations, fibers and metal particles has thoroughly investigated in this paper.

Key words: ceramics, hardness, strength, impact toughness, strengthening, structural ceramic.

Introduction

The use of powder metallurgy methods in the production of structural ceramic materials makes it possible to obtain components with unique structures and a complex set of properties. The key properties determining the quality of structural ceramic materials:

1. Hardness is a property of a material that depends on the hardness of its primary phase and its composition;
2. Strength is a property that depends on the material's structure, as well as on crystal size, phase composition, and the state of various substances at the phase boundaries;
3. Impact toughness – this property depends on the material's fracture energy and structure;

Ceramic materials with a fine-grained, heterogeneous structure exhibit higher impact toughness than those with a homogeneous structure. This is because internal stresses—intentionally induced within the ceramic structure—increase the energy required for fracture. These stresses facilitate the formation of micro-cracks that inhibit the propagation of primary cracks, thereby enhancing the material's resistance to cracking. Naturally, this mechanism helps prevent macroscopic failure of the material. In

structural ceramics, this effect is achieved by incorporating substances with a higher coefficient of thermal expansion than that of the base material. A prime example is the incorporation of titanium nitride (TiN) into silicon nitride (Si_3N_4) to improve fracture toughness; the coefficient of thermal expansion for TiN is significantly higher than that of Si_3N_4 . This outcome can also be achieved by introducing substances that undergo phase transformations during the firing process; adding zirconium dioxide to aluminum oxide to enhance its fracture toughness serves as an example of this approach.

Literature Review

X.L. Shi [5] investigated the effect of SiC on the mechanical properties of Al_2O_3 , finding that maximum strength (615 MPa) and hardness (25.2 GPa) are achieved at an SiC content of 20%. Zengbin Yin [6] proposed a composite containing nano- and micro-sized TiC powders as a material meeting the requirements for cutting tools.

Graphene nanoplatelets and carbon nanotubes: According to research by Jian Liu [7], the addition of 0.38% graphene nanoplatelets to Al_2O_3 resulted in increases of 30.78% in flexural strength and 27.20% in impact toughness, respectively.

TiC and Ti(CN). Zengbin Yin [74] proposed a composite containing nano- and micro-sized TiC powders as a material meeting the requirements for cutting tools, specifically highlighting high fracture toughness ($8.3 \text{ MPa}\cdot\text{m}^{1/2}$) and flexural strength (916 MPa). In this material, both the grain boundaries and the grains themselves are strengthened due to the presence of a plastic phase and nano-sized additives. A significant increase in fracture toughness ($7.4 \text{ MPa}\cdot\text{m}^{1/2}$) was observed upon adding Ti(CN) to Al_2O_3 ; this is attributed to the formation of a spatial network structure and strengthening mechanisms during sintering [81]. It has been shown that while variations in TiC particle size do not significantly affect hardness and fracture toughness, they have a substantial impact on flexural strength and density.

Research Methodology

Main mechanical properties of various advanced ceramic composites has been compared in terms of hardness, strength and impact toughness by studying main research results obtained by various researchers. The basic mechanical properties of the samples has been determined by using Brinell Rockwell and Vickers Hardness Tester

THBRV-187.5 equipments. The effect of strengthening mechanisms also has been found out via high-resolution microscopes.

Results and Discussion

In addition to modifiers, activating additives are incorporated into structural ceramic materials to enhance their densification and structural formation processes. Taking silicon nitride as an example, oxides of various metals serve this function; during the firing process, these activators interact with the silicon nitride to form phases such as sialons (solid solutions) or silicates at the grain boundaries. Furthermore, the resulting diffusion processes lead to the formation of oxynitride solid solutions, which facilitate mass transport and material densification. However, alongside their beneficial effects, these activating additives tend to reduce the hardness and refractoriness of the ceramic materials, as the resulting metal oxynitrides and silicate phases possess relatively low strength. Consequently, optimizing the composition of structural ceramic materials while balancing these factors remains one of the major challenges facing scientists today. Due to phase toughening resulting from reinforcement with ZrO_2 alone, impact toughness (4 MPa) and strength (600 MPa) increase fourfold and twofold, respectively (Figure 1) [3].

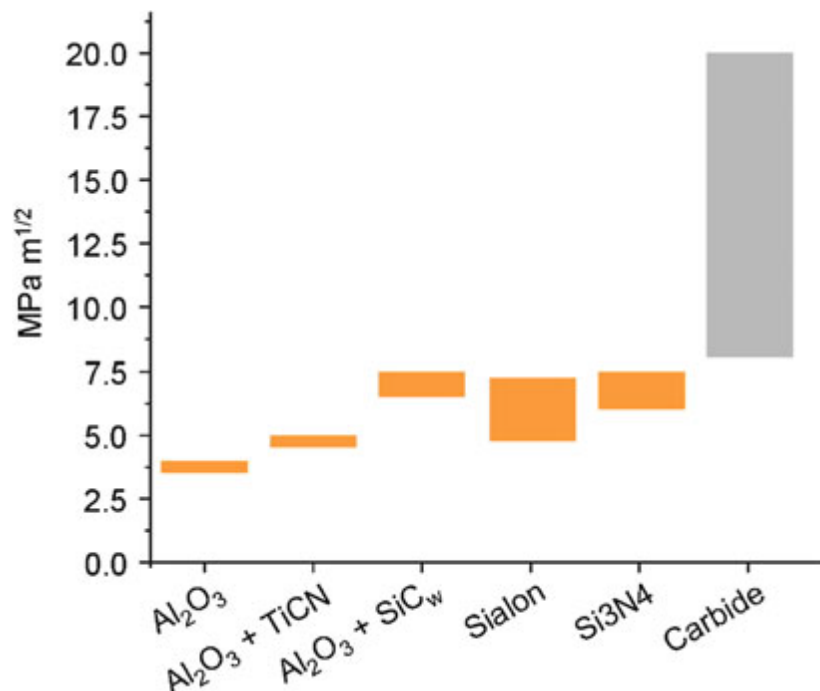


Figure 1. Impact toughness of ceramic materials.

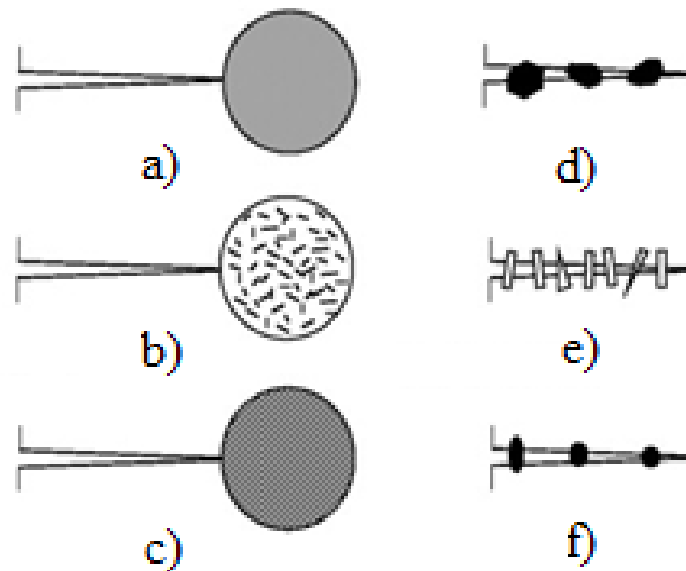


Figure 2. Strengthening mechanisms of Al_2O_3 . a – phase strengthening; b – strengthening via microcracks; c – via dislocations; d – via grains or brittle particles; e – strengthening with continuous fibers; f – dispersed metal particles.

The mechanisms widely used for the densification of alumina-based mineral-ceramic materials are illustrated in Figure 2 [4]. In addition to these densification mechanisms, research regarding the influence of alloying elements on the microstructure and physico-mechanical properties of alumina-based mineral-ceramic materials has been analyzed, with the effect of each alloying element examined individually.

Conclusion and Recommendations

To ensure the uniform distribution of activating additives, it is advisable to use aqueous solutions of the salts of the activating metals. Alternatively, the activating additives can be applied as a coating onto the surfaces of the particles forming the primary structural phases. This approach achieves maximum efficiency and allows for the use of minimal amounts of activating elements.

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APPLICATION OF ADVANCED CERAMIC COMPOSITES IN CUTTING TOOL INDUSTRY

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Annotatsiya: Keramik kompozit materiallarning o'ziga xos mexanik va yeyilishga chidamlilik xususiyatlari ularga bo'lgan talabning barqaror o'sishiga sabab bo'lmoqda hamda ularni kesuvchi asboblar, elektronika, aerokosmik va avtomobilsozlik kabi turli sohalarda qo'llash imkonini bermoqda. Ushbu maqolada keramik kompozit materiallarni ishlab chiqarish jarayonlari, ularga qo'yiladigan ekspluatatsion talablar va asosiy xususiyatlar tahlil qilinadi. Shuningdek, ushbu materiallar yordamida metallarga ishlov berishning asosiy jarayonlari haqida ma'lumotlar keltiriladi.

Kalit so'zlar: keramika, kesuvchi asbob, qattqlik, yeyilishga chidamlilik, mustahkamlik.

Аннотация: Уникальные механические и эксплуатационные характеристики (в частности, износостойкость) керамических композиционных материалов обуславливают неуклонный рост спроса на них и позволяют применять их в различных отраслях, таких как производство режущего инструмента, электроника, аэрокосмическая и автомобильная промышленность. В статье анализируются процессы изготовления, требования к эксплуатационным характеристикам и ключевые свойства керамических композиционных материалов. Также представлены сведения об основных процессах механической обработки металлов с использованием данных материалов.

Ключевые слова: керамика, режущий инструмент, твердость, износостойкость, прочность.

Abstract: The unique mechanical and wear properties of ceramic composite materials drive the steadily increasing demand for them, enabling their application across various sectors such as cutting tools, electronics, aerospace, and the automotive industry. This article analyzes the manufacturing processes, performance requirements, and key properties of ceramic composite materials. It also provides information on the primary processes involved in the machining of metals using these materials.

Key words: ceramic, cutting tool, hardness, wear resistance, strength.

Introduction

The cutting ability of a turning tool is determined by the physical and mechanical properties of the material from which it is made. The main properties that determine the tool's performance include hardness, heat resistance, wear resistance, thermal conductivity and adhesive ability. The hardness of the material from which the tool is made must exceed the hardness of the material being processed. Due to the fact that the working part of the tool is subject to significant cutting forces, creating bending deformations, the tool material must have strength. Modern hard alloys have high cutting properties due to the fact that they contain expensive and scarce metals - tungsten, titanium, cobalt. Of great importance is the creation of tool materials that, while having high heat resistance and wear resistance, would not contain such expensive elements [1].

Aluminum oxide is widely used in industry due to its high hardness, chemical inertness, and superior wear resistance, as well as its widespread availability compared to other ceramic materials. However, like all ceramic materials, the formation of cracks caused by internal stresses reduces impact toughness—a critical property for ceramics used in tooling applications. This limitation restricts the use of aluminum oxide in the tooling sector. Many researchers have investigated ways to enhance the mechanical properties of aluminum oxide and have proposed various strengthening mechanisms. Notably, improvements in properties have been achieved through the use of aluminum oxide-based composite materials, which involve incorporating a secondary phase—in

the form of powder or fibers—into the aluminum oxide matrix. This secondary phase may consist of metals, polymers, other ceramics, or refractory compounds [2].

Literature review

According to research conducted by Wilson Acchar [3], the addition of NbC results in increased impact toughness but decreased hardness. Conversely, the addition of Si₃N₄ leads to the formation of a core (Si₃N₄)–shell (Al₂O₃) structure, which enhances impact toughness (6.8 MPa). Increasing the Si₃N₄ content causes the formation of micro-cracks, agglomerates, and the β-Si₃N₄ phase at grain boundaries, thereby degrading mechanical properties [4]. Composites produced with the addition of B₄C exhibit mechanical properties inferior to those of pure Al₂O₃ [5]. The addition of WC, meanwhile, primarily ensures high hardness (26.41 GPa) [6].

Numerous studies have also been conducted on the effect of adding a metallic phase on the mechanical properties of alumina-based ceramic materials. According to research by the Chinese scientist Y.X. Li, an impact toughness of 6.5 MPa·m^{1/2} is achieved with a 5% Cu content; this is attributed to the fact that the release of O₂ gas from CuO during the sintering process creates voids that arrest crack propagation. Zengbin Yin observed a 41% increase in impact toughness resulting from the addition of metallic phases such as Ni and Co to Al₂O₃-based ceramic materials.

Research Methodology

The prepared ceramic plates were tested in the Metal Cutting Shop of the “Uzbek Technological Metals Combine” under the “Almalyk KMC” JSC, as well as by mechanical cutting and finishing of hardened steel parts installed on a lathe at the Department of Materials Science.

Results and Discussion

Ceramic composites for cutting tools are made in the form of aluminum oxide Al₂O₃ (alumina) inserts by pressing under high pressure followed by sintering. They have high hardness, temperature resistance (up to 1200°C), wear resistance and sufficient compressive strength. The disadvantages of these materials include high brittleness and low impact toughness. Tools equipped with mineral ceramics are usually

used for finishing during turning with a constant load and in the absence of vibration. The use of cutters with mechanical fastening of mineral-ceramic inserts instead of carbide cutters allows to increase the cutting speed by 2-3 times while simultaneously increasing the durability by 3-5 times. The use of such cutters is especially effective when processing hardened steel and cast iron, where the durability of carbide cutters is very low.

To equip cutters, mineral ceramics are used in the form of inserts of various shapes. Like cutters equipped with a hard alloy, cutters with mineral ceramic inserts are manufactured: 1) with inserts fixed in the holder; 2) with mechanical fastening; 3) with mechanical fastening in the holder of the insert, in which the insert is fixed.



Figure 1. Ceramic cutting tool. 1-carbide backing plate; 2-mineral ceramic insert; 3- insert fastening; 4- differential screw; 5-tool holder.

The most appropriate and widespread method is mechanical fastening of mineral-ceramic inserts. Soldering or gluing should be used only when this is determined by the design features of the tool, for example, for boring cutters, some multi-blade tools, etc. The main advantage of mechanical fastening is the ability to quickly change a dull or broken plate. In this case, most often there is no need to remove the cutter from the machine. Practice shows that with the correct distribution of the load on the insert and the appropriate arrangement of the supports, mechanical fastening ensures complete reliability from breakage. The presence of some freedom in the movement of the insert in the event of a deviation from the smoothness of the work during the cutting process (for example, with heterogeneity of the work piece) protects the insert from damage. For a soldered plate, due to the non-detachable fastening, this possibility disappears. The third method of fastening is a combination of the first two methods and, if used successfully, can ensure reliable operation of the cutter [2-3].

In order to determine the wear resistance of new powder composite mineraloceric plates, comparative tests were conducted based on the size of the worn surface and tests to determine the change in the weight of the plates before and after mechanical cutting and finishing (Figure 2).

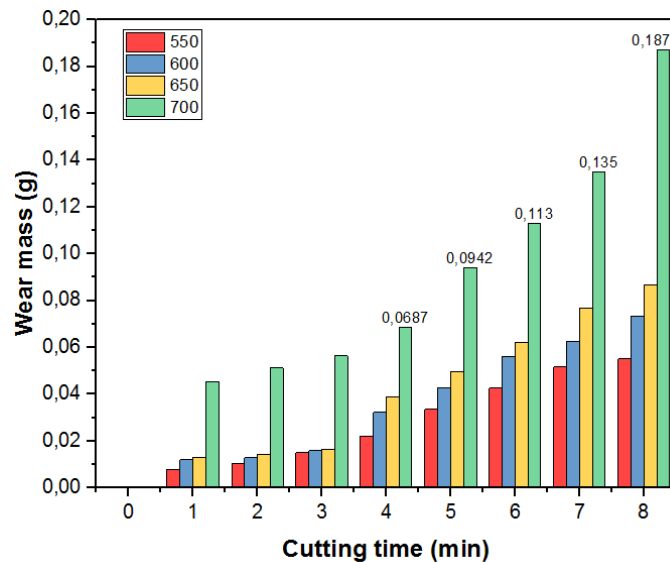


Figure 2. Deflection amount of ceramic composite inserts when different cutting speed (550, 600, 650 and 700) and feed rate 0.2

From the this graph, it can concluded that the wear rate of samples has show dramatic increase from 0,045 to 0,187 gr when the cutting speed was 700 m/min while this changes was minor consisting of the highest 0,05 gr for 550 m/min cutting speed.

Conclusion and Recommendations

The following conclusion has been drawn according to the above discussion and results:

- Due to frequent chipping of cutting edges, multi-faceted (mainly three- and four-faceted) inserts with the corresponding number of cutting edges are used in our and foreign practice.
- The economic feasibility of the widespread introduction of mineral ceramics instead of hard alloys is very convincingly emphasized by the following comparison: technical aluminum oxide costs 125 times less than tungsten carbide powder, which is the raw material for the production of hard alloys.

- However, for the widespread industrial use of mineral ceramics as a tool material, it is necessary to significantly increase its strength and ensure sufficient homogeneity of the physical, mechanical and cutting properties of ceramic inserts.

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VOCABULARY GROWTH IN UZBEK TEXTS: A REGRESSION STUDY

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Annotatsiya. Ushbu tadqiqotda o'zbek tilidagi matnlarda tokenlar soni bilan noyob tokenlar ulushi o'rtasidagi bog'liqlik o'rganilgan. Ishning maqsadi mazkur bog'liqlikning xususiyatlarini chiziqli model va Xips qonuniga asoslangan darajali model yordamida aniqlash hamda ularning tavsiflash aniqligini taqqoslashdan iborat. Tadqiqotning ilmiy ahamiyati o'zbek tilidagi matnlarda lug'at tarkibining o'sish qonuniyatlari bo'yicha empirik ma'lumotlar olinganligi bilan belgilanadi. Amaliy ahamiyati esa TF–IDF usulidan foydalanib matnlarni o'xshashlik bo'yicha taqqoslash, avtomatik referatlash, tasniflash va klasterlashga mo'ljallangan tabiiy

tilni qayta ishlash tizimlarini ishlab chiqishda olingan natijalardan foydalanish imkoniyati bilan izohlanadi. Tadqiqot davomida turli hajmdagi 132 ta o'zbek tilidagi matn tahlil qilinib, ulardagi tokenlar soni va noyob tokenlar ulushi aniqlangan hamda chiziqli va Xips qonuniga asoslangan darajali regressiya modellari qurilgan. Natijalar Xips qonuniga asoslangan darajali model o'rganilayotgan bog'liqlikni chiziqli modelga nisbatan aniqroq ifodalashini ko'rsatdi: modellarning moslik darajasi mos ravishda taxminan 94% va 69% ni tashkil etdi. Noyob tokenlar soni matn uzunligiga nisbatan sustroq, ya'ni subchiziqli sur'atda o'sishi aniqlandi. Olingan natijalardan o'zbek tilidagi matnlarni qayta ishlashda lug'at hajmi, TF-IDF vektorlarining o'lchami va tezkor xotiraga bo'lgan talabni oldindan baholashda foydalanish mumkin.

***Kalit so'zlar:** matn tahlili, regressiya tahlili, Xips qonuni, leksik xilma-xillik, o'zbek tili korpusi.*

Аннотация. Настоящее исследование посвящено изучению связи между числом токенов в узбекоязычных текстах и процентной долей уникальных токенов. Цель состоит в том, чтобы установить форму этой связи в рамках линейной модели и степенной модели типа закона Хипса и сравнить их описательную эффективность. Научная значимость исследования заключается в предоставлении эмпирически обоснованных данных об этой закономерности именно для узбекского языка, где она ранее не исследовалась в такой форме. Практическая мотивация обусловлена более широким диссертационным исследованием автора по сравнению текстов на схожесть, автоматическому реферированию, классификации и кластеризации текстов с использованием TF-IDF, размерность векторов которого определяется объёмом словаря и, следовательно, влияет на требования к памяти. В исследовании был проанализирован корпус из 132 узбекоязычных текстов разного объёма, вычислены число токенов и доля уникальных токенов, а связь между ними смоделирована с помощью линейной и степенной регрессии типа закона Хипса. Результаты показывают, что закон Хипса описывает эту связь значительно точнее линейной модели: степень

соответствия составила примерно 94% против 69% соответственно. Полученные результаты демонстрируют, что рост числа уникальных токенов является сублинейным, а не пропорциональным длине текста. Это создаёт основу для заблаговременного прогнозирования объёма словаря, размерности вектора и требований к памяти для приложений на основе TF-IDF, помогая предвидеть и смягчить проклятие размерности при обработке узбекоязычных текстов.

Ключевые слова: анализ текста, регрессионный анализ, закон Хинса, лексическое разнообразие, корпус узбекского языка.

Abstract. This study investigates the relationship between the number of tokens in Uzbek-language texts and the percentage share of unique tokens. The purpose is to determine the form of this relationship under a linear model and a Heaps'-law-type power-law model and to compare their descriptive performance. The scientific significance of the study lies in providing empirically grounded evidence on this regularity specifically for the Uzbek language, where it has not previously been examined in this form. The practical motivation arises from the author's broader dissertation research on text similarity, text summarisation, text classification, and text clustering using TF-IDF, whose vector dimensionality is determined by vocabulary size and therefore affects memory requirements. The study analysed a corpus of 132 Uzbek-language texts of different sizes, calculated token counts and unique-token shares, and modelled their relationship using linear and Heaps'-law-type power-law regression. The results show that Heaps' law describes the relationship considerably more accurately than the linear model, with goodness of fit of approximately 94% compared with 69%, respectively. The findings demonstrate that unique-token growth is sublinear rather than proportional to text length. This provides a basis for forecasting vocabulary size, vector dimensionality, and memory requirements of TF-IDF-based applications in advance, helping to anticipate and mitigate the curse of dimensionality in Uzbek-language text processing.

Key words: *text analysis, regression analysis, Heaps' law, lexical diversity, Uzbek language corpus.*

Introduction

A large share of practical natural-language-processing (NLP) work – comparing documents for similarity, automatically summarising a text, classifying documents into categories, and clustering documents into groups – is carried out using TF-IDF-based vector representations, in which every document is mapped to a vector whose length equals the size of the vocabulary observed in the collection. This representation is simple, interpretable, and effective, but it carries a structural risk: if vocabulary size were to grow essentially in step with text length, then TF-IDF vectors for realistically sized documents could become so high-dimensional that building, storing, and comparing them would be computationally impractical – an instance of the well-known curse of dimensionality. Whether this risk is merely theoretical or a genuine practical obstacle depends entirely on the actual mathematical relationship between text length and vocabulary size, which must be established empirically for the language and text types in question. This question is the starting point of the present study, which forms part of a broader programme of dissertation research on TF-IDF-based text similarity, summarisation, classification, and clustering for the Uzbek language: before such methods can be applied with confidence at scale, it is necessary to know, quantitatively, how the dimensionality of their underlying representations is expected to grow as texts grow longer.

The statistical study of natural-language texts has, for more than a century, revealed that many surface properties of language – word frequencies, sentence lengths, the growth of vocabulary with text length – are not arbitrary but obey remarkably stable quantitative laws. Two of these laws dominate the literature: Zipf's law, which states that the frequency of a word is approximately inversely proportional to its rank in a frequency-sorted list [1, 2], and Heaps' law (also called Herdan's law), which states that the size of the vocabulary V of a text or corpus grows sublinearly with its length N , typically as a power law $V \propto N^\beta$ with $0 < \beta < 1$ [1,4,5]. Although these two laws were discovered independently and in different contexts, a substantial body of theoretical

work has since shown that they are mathematically linked: under mild assumptions, a Zipfian frequency distribution with exponent close to unity implies a Heaps-type vocabulary-growth law, and vice versa [7–9]. Heaps’ law is the natural candidate hypothesis for resolving the question raised above, since it directly predicts whether vocabulary size – and hence TF-IDF vector dimensionality – grows proportionally with text length (which would make the curse of dimensionality a serious concern) or sublinearly (which would make it far more manageable in practice).

From an applied standpoint, Heaps’ law is far more than a linguistic curiosity. It underlies practical decisions in information retrieval (how large an inverted index must be for a collection of a given size) [2, 10], in corpus design (how many documents are needed before the vocabulary of a language variety is adequately sampled) [6, 7], and, increasingly, in natural language processing (NLP) engineering, where the dimensionality of a bag-of-words, one-hot, or TF-IDF representation is by construction equal to the vocabulary size V of the document or corpus being processed. Because many classical text-representation techniques (term–document matrices, co-occurrence matrices, naive Bayes and logistic-regression classifiers over sparse bag-of-words or TF-IDF features) have memory and computational costs that scale with V or even V^2 , and because V itself grows without bound as $N \rightarrow \infty$ (albeit, as this study will show, sublinearly), high-dimensional sparse text representations are a textbook illustration of the curse of dimensionality, a concept widely applied to machine learning and language modelling, where the number of possible n -grams over a vocabulary of size V grows as V^n [15, 16]. A predictive, empirically calibrated relationship between text length and vocabulary size therefore has direct engineering value for exactly the tasks motivating this study – text similarity, summarisation, classification, and clustering: it allows one to forecast, before a document or corpus is even fully processed, how large the resulting TF-IDF vector dimensionality, and hence the memory footprint of any similarity, summarisation, classification, or clustering pipeline built on it, is likely to be.

For high-resource languages such as English, Heaps’ law has been verified and re-verified on enormous corpora, from digitised book collections [6] to the Google Books Ngram corpus [11] to modern large-language-model-generated text [12], and its

connection to Zipf's law and to neural text generation continues to be studied [13,14]. For Turkic and, more specifically, for Central Asian languages, the picture is far less complete. Uzbek is officially classified as a low-resource language for NLP purposes: it is morphologically rich and agglutinative, and although a growing community of researchers has produced part-of-speech taggers [19], stop-word lists [22], semantic-similarity datasets [20], and text-classification corpora [21,24], quantitative-linguistic regularities such as Heaps' law have, to the best of our knowledge, not previously been estimated for Uzbek prose with a dedicated regression study. Related corpus-statistical work exists for other languages of the region and of the Commonwealth of Independent States (CIS): frequency-dictionary and corpus-statistics methodology has a long tradition in Russian and Kazakh quantitative linguistics [17, 18], and the broader mathematical relationship between high-dimensional data representations and statistical learning has likewise received sustained attention in the CIS scientific literature [16].

Literature Review

For clarity, it is useful to group the relevant literature into three strands that jointly motivate the present study. International (non-CIS) literature on Heaps' and Zipf's laws. The foundational statement of the vocabulary-growth law is due to Heaps [1], building on earlier observations by Herdan and Guiraud; its formal information-retrieval treatment is standard in modern textbooks [2, 10]. Contemporary work continues to refine both the empirical estimation and the theoretical status of the law: Chacoma and Zanette decompose vocabulary growth by grammatical class in a corpus of English literary works and show that the law holds well for the "multi-text" aggregate statistic even though within-text new-word appearance departs from a pure power law [4]; Font-Clos and Corral study the two variants of Heaps' law (single-document and multi-document) and their log-log convexity properties, reporting exponents broadly consistent with the one estimated here [5,11]; Petersen et al. use massive digitised book collections to show that the deceleration of new-word appearance reflects a genuine allometric scaling law rather than a sampling artefact [6]; and the mathematical equivalence between Zipf's rank-frequency law and Heaps' vocabulary-growth law has been rigorously derived under various assumptions on the tail of the frequency

distribution [7–9]. More recent work has extended Heaps’-law analysis to machine-generated text, showing that large-language-model output follows a Heaps-type law with an exponent that depends on model size and sampling temperature [12, 13], and to spoken as opposed to written language [14]. The engineering relevance of vocabulary size to representation dimensionality was made explicit for language modelling by Bengio et al., who identified the combinatorial explosion of possible n-grams over a vocabulary of size V as the central obstacle that neural, distributed word representations were designed to overcome [15]; more recent theoretical treatments continue to explore the relationship between high dimensionality, sparsity, and learnability [16]. CIS (Commonwealth of Independent States) and regional Turkic-language literature. Quantitative and corpus-statistical linguistics has an established tradition in the Russian- and Kazakh-language scientific literature, historically associated with frequency-dictionary compilation and, more recently, with the construction of large national text corpora. Contemporary examples include methodological work on the frequency and diachronic dynamics of language units using modern Russian-language corpora and web-based “natural corpora” [17], and the description of the National Corpus of the Kazakh Language and its use for author- and register-specific frequency analysis [18]. On the mathematical side, the treatment of high-dimensional data representations and the tension between the curse and the “blessing” of dimensionality has likewise been actively developed within the CIS applied-mathematics community [16]. These strands provide both a methodological precedent and a regional scientific context for the present, Uzbek-language-specific study. Uzbekistan-specific NLP and corpus-resource literature. Because Uzbek is a low-resource, agglutinative Turkic language, most of the directly relevant prior work has focused on building foundational NLP resources rather than on quantitative-linguistic law estimation as such. This includes rule-based part-of-speech tagging [19], morphologically and syntactically tagged corpora, semantic-similarity and relatedness evaluation datasets [20], multi-class news-text classification [21], and, closest to the present work, the author’s own prior contributions on Uzbek stop-word detection [22], the construction and TF-IDF-based analysis of the School Corpus of Uzbek primary-school textbooks [23], unsupervised clustering and rule-based

labelling of Uzbek educational texts [25], grade-level classification of Uzbek literary texts by complexity features [26], base-word extraction from the School Corpus [27,28], and TF-IDF-based classification of Uzbek educational texts [24]. None of this prior

work, including the author's own, has previously estimated a Heaps'-law exponent for Uzbek text, which is the specific gap addressed here. The present study fills this gap for the Uzbek language. Building on a corpus of 132 Uzbek-language texts of substantially different lengths and genres – ranging from short (127-word) excerpts to long (79,531-word) works, and including school-textbook material previously assembled and analysed by the author and colleagues in the context of the “School Corpus” project [23,27] – we hypothesise that a systematic relationship exists between the total number of tokens N in an Uzbek text and the percentage share of its unique tokens $P = 100V / N$, and we test this hypothesis using two competing methods (a linear regression and a Heaps'-law-type power-law regression) in order to answer the following research questions:

(i) Is the percentage share of unique words $P = 100V / N$ linearly related to the total word count N , and if so, how strong and how statistically significant is that linear relationship?

(ii) Does a power-law (Heaps-type) model describe the same relationship substantially better than the linear model, in terms of both explained variance and residual error, thereby confirming Heaps' law as the more appropriate of the two methods?

(iii) What are the corresponding growth law and qualitative properties (monotonicity, concavity, asymptotic behaviour) of the absolute number of unique words $V(N)$ implied by the best-fitting model of $P(N)$, and do they confirm that unique-token count does not grow out of proportion to token count even for very long texts?

(iv) To what extent does this growth law imply that TF-IDF-based text-similarity, summarisation, classification, and clustering pipelines are exposed to the curse of dimensionality for texts of realistic, up-to-book-length size, and what vector dimensionality and memory footprint should be expected for a document of the size of a typical book?

The remainder of the paper is organised as follows. Section 2 (Material and Methods) formalises the problem, describes the corpus, and specifies the two competing regression models together with the statistical criteria used to compare them. Section 3 (Results and Discussion) reports the estimated coefficients, correlation and determination statistics, a formal theorem establishing the qualitative behaviour of the fitted models and of the implied vocabulary-growth law, an asymptotic analysis of both models, an interpretation of the results in the light of the international and CIS literature on Heaps' law, a numerical illustration of the memory-estimation application, and a discussion of the limitations of the study. Section 4 (Conclusion) summarises the main findings and outlines directions for future work.

Research Methodology

Problem statement and notation: Let a corpus consist of n texts, and let the i -th text ($i = 1, 2, \dots, n$) contain N_i word tokens in total, of which V_i are unique word types (i.e., V_i is the size of the set of distinct word forms occurring in the i -th text). We define the percentage share of unique words in the i -th text as

$$P_i = 100 \frac{V_i}{N_i}, \quad i = 1, \dots, n \quad (1)$$

so that the data set of observations used throughout this study is

$$S = \{(N_i, P_i)\}_{i=1}^n, \quad n = 132 \quad (2)$$

Given any Uzbek-language text of total length N , we hypothesise that its number of unique tokens V (equivalently, its percentage share of unique tokens $P = 100V/N$) is not independent of N but is related to it by a systematic, estimable functional law. Testing this hypothesis, and identifying the correct functional form of the relationship, is the central empirical task of this study, because the correct functional form directly determines how the dimensionality of any TF-IDF-based representation of the text is

expected to behave as N grows. Two competing methods. We evaluate this hypothesis using two competing regression methods, specified in full in Sections 2.4–2.7 below:

(a) a linear regression method, which models P as an affine (straight-line) function of N ; and

(b) a power-law regression method of the Heaps'-law type, which models the absolute vocabulary size as $V(N) = AN^\beta$ for constants $A > 0$ and $0 < \beta < 1$ – equivalently, $P(N) = 100AN^{\beta-1}$

– reflecting the classical empirical observation, first formalised by Heaps [1], that the number of distinct word types in a text grows sublinearly (i.e., strictly slower than proportionally) with the number of word tokens.

The two methods embody two qualitatively different predictions about the same hypothesis: the linear method allows P (and hence, implicitly, V) to change with N at a constant absolute rate, with no built-in guarantee that vocabulary growth remains sublinear, whereas the Heaps-law method builds sublinear growth into the model by construction. Comparing their fit to the data (Sections 3.1–3.3) therefore serves the dual purpose of (i) testing whether the hypothesised N – V relationship exists at all, and (ii) determining which of the two candidate functional forms correctly describes it – which, as Section 3 will show, is decisively the Heaps-law method. The overall research objective is to determine, on the basis of S , the functional law by which P changes as N increases, and to build a validated regression model for this dependence.

Corpus and data collection

The empirical basis of the study is a corpus of $n = 132$ Uzbek-language texts of heterogeneous origin and length. The corpus combines (a) short thematically organised excerpts drawn from school-subject materials (labelled by section and lesson number, e.g. “1_1_1_bolim.txt”), consistent with the compilation methodology of the previously published School Corpus of Uzbek primary-school textbooks [23,27], and (b) substantially longer continuous texts of literary and expository prose (labelled, e.g., “onat1.txt”, “uqish2.txt”, “adab5.txt”), used here specifically to extend the length range of the sample and thereby to make the estimation of the vocabulary-growth exponent

more reliable. Text length in the resulting corpus spans almost three orders of magnitude, from $N_{\min} = 127$ to $N_{\max} = 79,531$ word tokens (total corpus size $\sum_i N_i = 973,931$ tokens), which is essential for reliably distinguishing a linear from a power-law relationship: over a narrow range of N any smooth, monotonically decreasing function of N can be locally approximated by a straight line, so a wide dynamic range in N is what allows the two competing hypotheses to be statistically discriminated. For every text, tokenisation was performed at the word level (space- and punctuation-delimited), and V_i was computed as the cardinality of the set of distinct word tokens after case normalisation, without additional lemmatisation or stemming, so that V_i reflects surface word-form diversity rather than lexeme-level diversity. This is a deliberate methodological choice: because Uzbek is an agglutinative language in which a single lexeme can surface as dozens of distinct inflected and derived word forms, surface-form vocabulary size is the quantity that is directly relevant to the memory footprint of word-form-based NLP representations (bag-of-words, one-hot encodings, n-gram language models), which is precisely the applied question motivating this study. Table 1 shows a sample of the underlying per-text data table – the beginning, middle, and end of the 132 rows – illustrating the two raw counts, N_i and V_i , from which every subsequent statistic in this paper is derived.

The full corpus underlying Table 1 – the raw plain-text files for all 132 Uzbek-language documents from which every count and statistic in this paper is derived – has been deposited in the Zenodo repository and is openly available under a Creative Commons Attribution 4.0 licence [29], allowing the reported token counts, unique-word counts, and all downstream regression results to be independently reproduced and extended.

Descriptive statistics of the corpus:

Table 2. summarises the distribution of the three key variables – total word count N , unique-word count V , and percentage share of unique words P – across the $n = 132$ texts of the corpus. The distributions of N and V are strongly right-skewed, as is typical of text-length distributions in mixed-genre corpora: the median text length (1,546.5 tokens) is far below the mean (7,378.3 tokens), reflecting the fact that the majority of

the corpus (105 of 132 texts, i.e., about 80%) consists of relatively short school-subject excerpts of fewer than 3,000 tokens, while a smaller number of long literary and expository texts (up to 79,531 tokens) extend the upper tail of the distribution. This heterogeneity is deliberate: as explained in Section 2.2, the wide dynamic range of N is precisely what allows a power-law relationship to be statistically distinguished from a linear one, since the two functional forms are difficult to tell apart when N varies over a narrow range.

Table 1. Sample rows from the per-text data table ($n = 132$ texts)

File name	Total tokens N_i	Unique types V_i	P_i (%)
3_2_13_bolim.txt	127	119	93.70
2_4_11_bolim.txt	230	190	82.61
1_1_12_bolim.txt	484	380	78.51
1_2_15_bolim.txt	640	513	80.16
2_4_9_bolim.txt	855	574	67.13
...	...	...	...
2_2_10_bolim.txt	1,529	1,020	66.71
3_1_7_bolim.txt	1,543	935	60.60
3_1_13_bolim.txt	1,550	988	63.74
2_2_6_bolim.txt	1,562	993	63.57
3_1_5_bolim.txt	1,582	1,061	67.07
...	...	...	...
adab112.txt	41,739	15,584	37.34
adab5.txt	45,044	15,649	34.74
adab7.txt	65,126	21,130	32.44
adab8.txt	67,566	19,976	29.57
adab9.txt	79,531	24,674	31.02

Table 2. Descriptive statistics of the corpus (n = 132 texts)

Variable	Min	Median	Mean	Max	Std. dev.
N (total tokens)	127	1,546.5	7,378.3	79,531	14,326.5
V (unique types)	119	1,005.0	2,923.6	24,674	4,625.4
P (% unique words)	29.57	63.84	59.82	93.70	12.61

Linear regression model

The relationship was first examined using the classical simple linear regression model

$$P_i = a + bN_i + \varepsilon_i, \quad i = 1, \dots, n \quad (3)$$

where ε_i are assumed to be independent, zero-mean disturbances. By the ordinary-least-squares (OLS) principle, the slope and intercept are estimated as

$$\hat{b} = \frac{\sum_{i=1}^n (N_i - \bar{N})(P_i - \bar{P})}{\sum_{i=1}^n (N_i - \bar{N})^2} \quad (4)$$

$$\hat{a} = \bar{P} - \hat{b} \bar{N} \quad (5)$$

where $\bar{N} = n^{-1} \sum_{i=1}^n N_i$ and $\bar{P} = n^{-1} \sum_{i=1}^n P_i$, with $n = 132$.

Pearson correlation coefficient

The strength of the linear association between N and P was quantified by the Pearson product– moment correlation coefficient

$$r = \frac{\sum_{i=1}^n (N_i - \bar{N})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^n (N_i - \bar{N})^2} \sqrt{\sum_{i=1}^n (P_i - \bar{P})^2}} \quad (6)$$

together with the associated two-sided significance test (under the null hypothesis $\rho = 0$) based on the t-statistic

$$t_r = r \sqrt{\frac{n-2}{1-r^2}}, \quad \text{df} = n - 2 \quad (7)$$

Goodness of fit: coefficient of determination and error measures

For the linear model, the coefficient of determination is

$$R_L^2 = 1 - \frac{\sum_{i=1}^n (P_i - \hat{P}_{L,i})^2}{\sum_{i=1}^n (P_i - \bar{P})^2} \quad (8)$$

where $\hat{P}_{L,i} = \hat{a} + \hat{b}N_i$, and the associated mean squared error and root mean squared error are

$$\text{MSE}_L = \frac{1}{n} \sum_{i=1}^n (P_i - \hat{P}_{L,i})^2, \quad \text{RMSE}_L = \sqrt{\text{MSE}_L} \quad (9)$$

To test the overall significance of the linear regression, the F-statistic

$$F_L = \frac{(\text{SST} - \text{SSE}_L)/1}{\text{SSE}_L/(n-2)}, \quad (10)$$

$$\text{SST} = \sum_{i=1}^n (P_i - \bar{P})^2, \quad \text{SSE}_L = \sum_{i=1}^n (P_i - \hat{P}_{L,i})^2$$

was computed and compared against the $F_{1,n-2}$ distribution.

Power-law (Heaps) regression model

Because a scatter plot of the empirical pairs (N_i, P_i) shows pronounced curvature – a steep decline for small N followed by a slowly flattening tail for large N – a power-law model of the Heaps type was fitted. Using Table 1, for each i -th document we again write the percentage share of unique words as

$$P_i = \frac{V_i}{N_i} \times 100, \quad i = 1, \dots, 132 \quad (11)$$

where, as before, N_i is the total number of word tokens in the i -th document and V_i is the number of unique (non-repeating) word tokens in it. According to Heaps' law, this relationship takes the form of a power function, $V_i(N_i) = AN_i^\beta$. Substituting this expression into (11), we get the following expression:

$$P_i(N_i) = 100AN_i^{\beta-1}, \quad A > 0, \quad 0 < \beta < 1 \quad (12)$$

Taking the natural logarithm of both sides of (12) gives

$$\ln P_i(N_i) = \ln(100A) + (\beta - 1)\ln N_i \quad (13)$$

Introducing the notation

$$y_i = \ln P_i(N_i), \quad \alpha = \ln(100A), \quad x_i = \ln N_i, \quad h = \beta - 1 \quad (14)$$

For consistent notation, write $C = 100A = e^\alpha$, so that $P(N) = CN^h$ and $V(N) = AN^\beta$, where $\beta = 1 + h$.

equation (13) becomes the linear relationship

$$y_i = \alpha + hx_i \quad (15)$$

which is fitted as $y_i = \alpha + hx_i + \varepsilon_i$. Consider the sum-of-squares function

$$f(\alpha, h) = \sum_{i=1}^{132} (hx_i + \alpha - y_i)^2 \quad (16)$$

Applying the method of least squares, we find the extremum of $f(\alpha, h)$ by setting its partial derivatives to zero:

$$\frac{\partial f}{\partial h} = 0, \quad \frac{\partial f}{\partial \alpha} = 0 \quad (17)$$

which gives the system of equations

$$\sum_{i=1}^{132} (hx_i + \alpha - y_i) x_i = 0, \quad \sum_{i=1}^{132} (hx_i + \alpha - y_i) = 0 \quad (18)$$

Expanding the brackets and grouping terms yields the normal equations

$$\begin{aligned} h \sum_{i=1}^{132} x_i^2 + \alpha \sum_{i=1}^{132} x_i &= \sum_{i=1}^{132} x_i y_i, \\ h \sum_{i=1}^{132} x_i + 132\alpha &= \sum_{i=1}^{132} y_i \end{aligned} \quad (19)$$

Introducing the means $\bar{x} = \frac{1}{132} \sum_{i=1}^{132} x_i$ and $\bar{y} = \frac{1}{132} \sum_{i=1}^{132} y_i$, and eliminating α between the two equations of (19), we obtain

$$h(\sum_{i=1}^{132} x_i^2 - 132\bar{x}^2) = \sum_{i=1}^{132} x_i y_i - 132\bar{x}\bar{y} \quad (20)$$

from which the final formulas for h and α follow:

$$h = \frac{\sum_{i=1}^{132} x_i y_i - \bar{y} \sum_{i=1}^{132} x_i}{\sum_{i=1}^{132} x_i^2 - \bar{x} \sum_{i=1}^{132} x_i}, \quad \alpha = \bar{y} - h\bar{x} \quad (21)$$

For the present corpus of 132 documents, these formulas give

$$\hat{h} = -0.19543, \quad \hat{\alpha} = 5.59494, \quad \hat{A} = 0.01e^{5.59494}, \quad \hat{\beta} = 1 + \hat{h} \quad (22)$$

so that (15) becomes

$$y = 5.59494 - 0.19543x \quad (23)$$

Substituting back into $P(N) = e^{\alpha} N^h$ gives

$$P(N) = e^{5.59494} N^{-0.19543} \quad (24)$$

that is,

$$P(N) = 269.06N^{-0.19543} \quad (25)$$

consistent with the estimate reported in Section 3.2. Two coefficients of determination are reported for the power-law model: the logarithmic-scale $R_{\log}^2$, obtained directly from the linear regression (13) on $(\ln N_i, \ln P_i)$, and the original-scale R_H^2 , computed by transforming the fitted values back to the original scale, $\hat{P}_{H,i} = 100\hat{A}N_i^{\hat{\beta}-1} = \hat{C}N_i^{\hat{h}}$, and evaluating

$$R_H^2 = 1 - \frac{\sum_{i=1}^n (P_i - \hat{P}_{H,i})^2}{\sum_{i=1}^n (P_i - \bar{P})^2}, \quad (26)$$

$$\text{MSE}_H = \frac{1}{n} \sum_{i=1}^n (P_i - \hat{P}_{H,i})^2, \quad \text{RMSE}_H = \sqrt{\text{MSE}_H}$$

Reporting R_H^2 on the original scale (rather than relying on $R_{\log}^2$ alone) is methodologically important, because a model that fits well in log–log coordinates does not automatically fit equally well after back-transformation; both statistics are therefore given side by side in Section 3.

Model-selection criteria. Because the linear and power-law models are non-nested, their comparison cannot rely on a nested-model F-test; instead, both models were compared on the common (original) scale of P using (i) R^2 , (ii) MSE/RMSE, and (iii) the Akaike and Bayesian information criteria,

$$\text{AIC} = n \ln \left(\frac{\text{SSE}}{n} \right) + 2k, \quad \text{BIC} = n \ln \left(\frac{\text{SSE}}{n} \right) + k \ln n \quad (27)$$

with $k = 2$ estimated parameters for both models, computed on the original scale of P so that the two models are directly comparable. Lower values of AIC and BIC indicate a better trade-off between goodness of fit and model complexity.

Software and reproducibility

All statistics reported below were computed in Python 3 using NumPy, SciPy, and statsmodels, applied directly to the tabulated (N_i, V_i, P_i) triples for all $n = 132$ texts. Every numerical value quoted in Sections 3–4 (including the confidence intervals and hypothesis-test statistics reported in Section 3, which extend the original point estimates) was independently recomputed from the raw corpus table as part of the preparation of this manuscript, and no figure has been taken on faith from an intermediate spreadsheet computation.

Results and Discussion

Linear regression results. Applying (4)–(5) to the empirical data set S ($n = 132$) gives

$$\hat{a} = 65.215196, \quad \hat{b} = -0.000731458 \quad (28)$$

so that the fitted linear regression equation is

$$\hat{P}_L(N) = 65.215196 - 0.000731458N \quad (29)$$

The 95% confidence intervals for the two coefficients are $\hat{a} \in [63.852, 66.578]$ and $\hat{b} \in [-0.000816, -0.000647]$; the corresponding t-statistics are $t_{\hat{a}} = 94.64$ and $t_{\hat{b}} = -17.05$ (both with 130 degrees of freedom, $p < 10^{-30}$), confirming that both coefficients are estimated with high precision and that the negative slope is highly statistically significant. Since $\hat{b} < 0$, the linear model indicates that the percentage share of unique words decreases as text length increases.

Pearson correlation coefficient

The Pearson correlation coefficient between N and P, computed by (6), is

$$r = -0.831333 \quad (30)$$

The associated two-sided test statistic is $t_r = -17.05$ with 130 degrees of freedom, giving

$$p \approx 5.75 \times 10^{-35} \quad (31)$$

so the negative linear association between total word count and the percentage share of unique words is strong and statistically significant at any conventional significance level.

Coefficient of determination and error of the linear model

The coefficient of determination (8) of the linear model is

$$R_L^2 = 0.691114 \quad (32)$$

i.e., approximately 69.11% of the total variance in P is explained by the linear model. The overall F-test of the regression (10) gives $F_L(1, 130) = 290.87$ ($p < 10^{-30}$), confirming that the linear model, taken on its own terms, is highly statistically significant. The mean squared error and root mean squared error of the linear model are

$$\text{MSE}_L = 48.708482, \quad \text{RMSE}_L = \sqrt{\text{MSE}_L} = 6.979146 \quad (33)$$

Power-law (Heaps) regression results

The structure of the empirical points shows that the decline of P with N is not linear: it is steep for small texts and becomes progressively flatter for larger ones – exactly the qualitative signature of a power law rather than a straight line. Fitting the log-linearised model (13) by ordinary least squares gives

$$\hat{C} = 269.061418, \quad \hat{h} = -0.195427, \quad \hat{A} = 2.69061418, \quad \hat{\beta} = 0.804573 \quad (34)$$

so that the fitted power-law regression model is

$$\hat{P}_H(N) = 269.061418N^{-0.195427} \quad (35)$$

The 95% confidence interval for the share exponent is $\hat{h} \in [-0.2035, -0.1873]$ (with $t_{\hat{h}} = -47.71$,

$p < 10^{-30}$), and for the share amplitude, $\hat{C} \in [252.31, 286.93]$; both parameters are therefore estimated with considerably higher relative precision than the corresponding linear-model coefficients.

The original-scale and logarithmic-scale coefficients of determination are

$$R_H^2 = 0.941920, \quad R_{\log}^2 = 0.945982 \quad (36)$$

and the corresponding error measures are

$$MSE_H = 9.158715, \quad RMSE_H = \sqrt{MSE_H} = 3.026337 \quad (37)$$

Comparative analysis of the two models

Table 3 summarises the coefficients and goodness-of-fit statistics of both models on a common footing (all error measures computed on the original, untransformed scale of P), and Figure 1 shows both fitted curves against the empirical data.

Table 3. Comparative statistical characteristics of the linear and power-law models

Indicator	Linear model	Power-law model
Empirical equation	$65.215196 - 0.000731458N$	$269.061418N^{-0.195427}$
Pearson r	-0.831333	—
R^2 (original scale)	0.691114	0.941920
$R^2_{\log}$ (log-log scale)	—	0.945982
MSE	48.708482	9.158715
RMSE	6.979146	3.026337
AIC (k = 2)	516.93	296.34
BIC (k = 2)	522.70	302.11

For the power-law model,

$$R_H^2 - R_L^2 = 0.941920 - 0.691114 = 0.250806 \quad (38)$$

i.e., the power-law model explains an additional 25.08 percentage points of the total variance in P relative to the linear model, and $MSE_H < MSE_L$ by a factor of more than five. The AIC and BIC values in Table 3, computed on the identical scale for both models, differ by more than 220 points in favour of the power-law model – a difference so large that, under standard AIC-based model-weight interpretation, the linear model

has, for all practical purposes, zero relative likelihood of being the better description of the data. All three independent lines of evidence (variance explained, residual error, and information criteria) therefore converge on the same conclusion: the power-law (Heaps) model describes the empirical relationship between text length and the percentage share of unique words in this corpus of Uzbek texts substantially more accurately than the linear model.

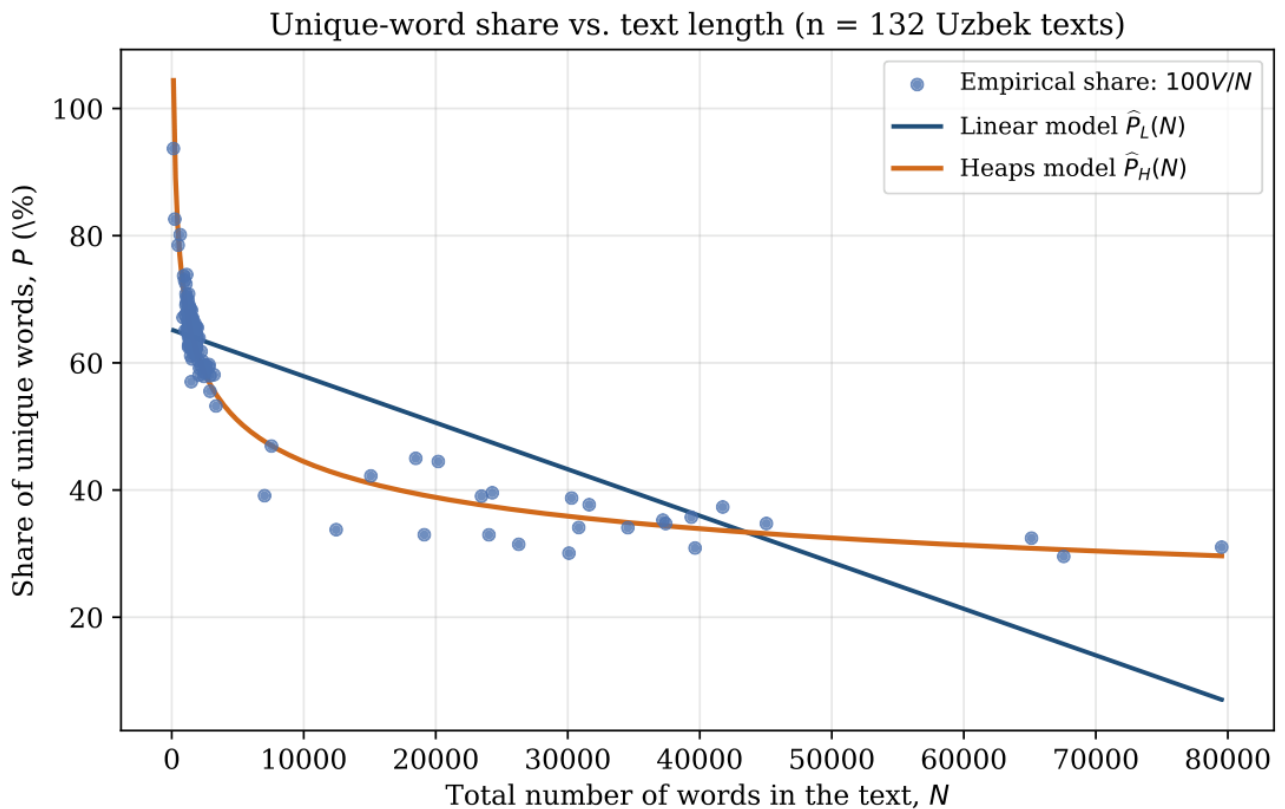


Figure 1. Empirical unique-word share $P = 100V/N$ for 132 Uzbek texts with the fitted linear and power-law models.

Qualitative properties of the fitted models. Statement 1 (Properties of the fitted power-law model). Let the corpus $S = \{(N_i, P_i)\}_{i=1}^{132}$ be as defined in (2), and let $\hat{P}_L(N)$ and $\hat{P}_H(N)$ be the linear and power-law regression models fitted in (29) and (35), respectively. Then:

- (a) the power-law model describes the observed data more accurately than the linear model, in the sense that $R_H^2 > R_L^2$ and $MSE_H < MSE_L$;
- (b) the function $\hat{P}_H(N)$ is strictly decreasing and strictly positive for all $N > 0$, with $\hat{P}_H(N) \rightarrow 0$ as $N \rightarrow \infty$;

(c) the absolute number of unique words implied by $\hat{P}_H(N)$ under the identity $P(N) = 100 V(N)/N$ is

$$V(N) = 2.69061418N^{0.804573} \quad (39)$$

and $V(N)$ is strictly increasing and strictly concave on $(0, \infty)$, i.e., the vocabulary grows without bound but at a rate that is strictly slower than the growth of the text itself (a Heaps-type sublinear growth law).

Proof. (a) Fitting the linear model to the empirical data gives, as shown in Section 3.1,

$$\hat{P}_L(N) = 65.215196 - 0.000731458N, \quad R_L^2 = 0.691114, \quad \text{MSE}_L = 48.708482.$$

while fitting the power-law model gives, as shown in Section 3.2,

$$\hat{P}_H(N) = 269.061418N^{-0.195427}, \quad R_H^2 = 0.941920, \quad \text{MSE}_H = 9.158715.$$

Comparing the two pairs of statistics directly gives $R_H^2 > R_L^2$ and $\text{MSE}_H < \text{MSE}_L$, which establishes part (a). (b) Differentiating (35) with respect to N gives

$$\hat{P}'_H(N) = 269.061418(-0.195427)N^{-1.195427} \quad (40)$$

For every $N > 0$ we have $N^{-1.195427} > 0$, and since $-0.195427 < 0$, it follows that

$$\hat{P}'_H(N) < 0 \quad \text{for all } N > 0 \quad (41)$$

so $\hat{P}_H$ is strictly decreasing on $(0, \infty)$. Positivity, $\hat{P}_H(N) = 269.061418 N^{-0.195427} > 0$ for all $N > 0$, is immediate because $269.061418 > 0$ and $N^{-0.195427} > 0$ for $N > 0$. Finally, because the exponent -0.195427 is negative, $N^{-0.195427} \rightarrow 0$ as $N \rightarrow \infty$, and hence $\hat{P}_H(N) \rightarrow 0$ as $N \rightarrow \infty$, which proves part (b). (c) By definition, $P(N) = 100 V(N)/N$, so

$$V(N) = \frac{NP(N)}{100} \quad (42)$$

Substituting the fitted power-law model (35) for $P(N)$ gives

$$V(N) = \frac{N}{100} 269.061418N^{-0.195427} = 2.69061418N^{1-0.195427} \quad (43)$$

$$V(N) = 2.69061418N^{0.804573} \quad (44)$$

Differentiating (44),

$$V'(N) = 2.69061418 \cdot 0.804573N^{-0.195427} > 0 \quad (N > 0) \quad (45)$$

since both factors are positive; hence $V(N)$ is strictly increasing on $(0, \infty)$.

Differentiating once more,

$$V''(N) = 2.69061418 \cdot 0.804573(-0.195427)N^{-1.195427} < 0 \quad (N > 0) \quad (46)$$

because the constant multiplying $N^{-1.195427}$ is negative while $N^{-1.195427} > 0$; hence $V(N)$ is strictly concave on $(0, \infty)$. Because the exponent in (44) satisfies

$$0 < 0.804573 < 1 \quad (47)$$

$V(N)$ grows strictly slower than N itself (formally, $V(N)/N \rightarrow 0$ as $N \rightarrow \infty$, which follows directly from part (b) since $V(N)/N = P(N)/100 \rightarrow 0$). This establishes that the absolute number of unique words increases without bound as the text grows, but its growth rate is strictly sublinear in N – the defining qualitative property of a Heaps-type vocabulary-growth law. The Statement is proved.

Corollary 2. Part (c) of Statement 1 resolves an apparent tension in the data: although the relative share of unique words $P(N)$ decreases with text length (part (b)), this does not mean that longer texts contain, in absolute terms, fewer distinct words – quite the opposite. Equation (39) shows that the absolute vocabulary size $V(N)$ is a strictly increasing, concave function of N , reflecting a positive but diminishing expected marginal vocabulary increment as text length grows. Figure 2 illustrates the resulting sublinear growth curve of $V(N)$ against the reference line $V = N$ (which would correspond to a text in which every single word token is distinct).

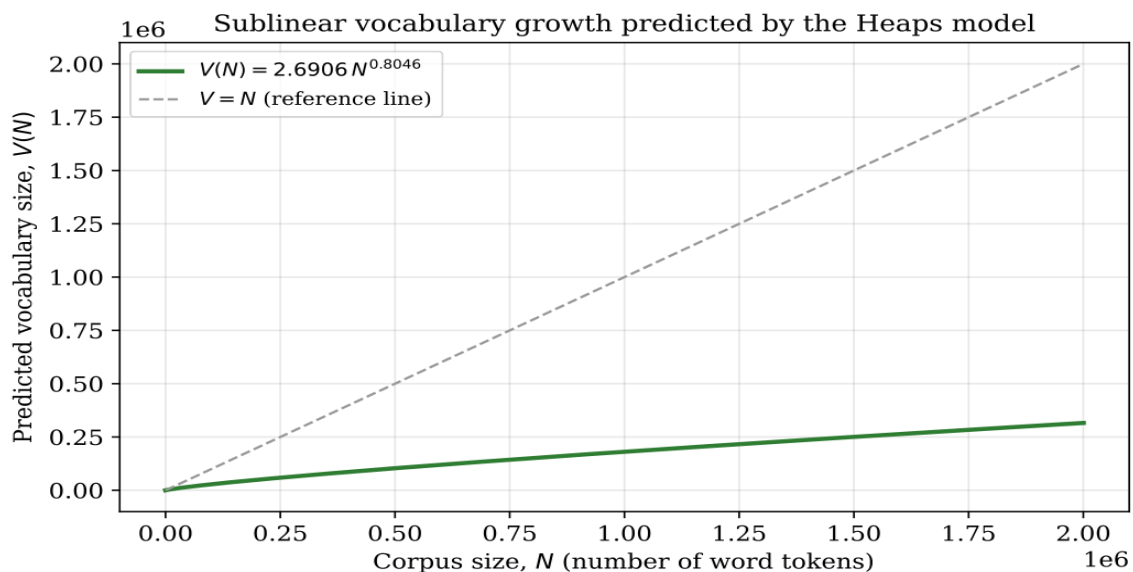


Figure 2. Predicted vocabulary size $V(N) = 2.69061418N^{0.804573}$ compared with $V = N$.

Application: forecasting vocabulary size and memory requirements

The applied motivation of this study is to convert the fitted vocabulary-growth law (39) into a usable engineering tool. Many classical and still widely used text-representation techniques in NLP construct a data structure whose size is directly determined by the vocabulary size V of the corpus being processed:

- a one-hot or bag-of-words representation of a single document requires a vector of dimension V ;
- a term-term co-occurrence matrix (used, e.g., to train classical count-based word-vector models) requires on the order of V^2 entries in the dense case;
- a dense embedding table for a vocabulary of size V with embedding dimension d requires $V \times d$ parameters.

Because V itself grows with corpus size according to (39), all three of these quantities can be estimated in advance, without first tokenising and indexing the full corpus, simply by substituting the intended corpus size N into $V(N) = 2.69061418N^{0.804573}$. Table 4 illustrates this with several representative corpus sizes, reporting the model-predicted vocabulary size $V(N)$, the memory required for a 300-dimensional dense embedding table stored in single-precision (4-byte) floating point, and the memory that would be required for a full dense $V \times V$ co-occurrence matrix at the same precision. The final row corresponds to $N = 79,531$, the length of the longest text in the present corpus, included for calibration.

Table 4. Illustrative memory-footprint estimates derived from the fitted vocabulary-growth model

Corpus size N	Predicted $V(N)$	Embedding table 300-dim., float32	Dense co-occurrence matrix, float32
10,000	4,448	5.3 MB	79 MB
79,531	23,588	28.3 MB	2.23 GB
100,000	28,361	34.0 MB	3.22 GB
1,000,000	180,838	217.0 MB	130.8 GB
10,000,000	1,153,088	1.38 GB	5,318 GB

Table 4. makes the practical relevance of the sublinear exponent concrete: because $\beta = 0.804573 < 1$, an embedding-table representation (whose size scales linearly in V , and hence as $N^{0.8046}$) grows considerably more slowly than the corpus itself, and a system designer can budget for it with reasonable confidence using (39) alone. A dense

co-occurrence-matrix representation, however, scales as $V^2 \propto N^{1.609}$ – faster than linearly in N despite the sublinear growth of V itself – and rapidly becomes intractable (note the jump from megabytes to terabytes between $N = 10^5$ and $N = 10^7$ in Table 4). This is precisely the curse-of-dimensionality phenomenon referred to in the Introduction [15, 16]: even a sublinearly growing vocabulary produces a quadratically (or, for n -gram models, exponentially, as V^n [15]) growing representation once that vocabulary is used as the dimensionality of a joint or pairwise structure.

Conclusion and Recommendations

Based on empirical data from 132 Uzbek-language texts, this study established the following results:

1. There is a strong negative linear correlation between the total number of words in a text and the percentage share of unique words:

$$r = -0.831333, \quad p \approx 5.75 \times 10^{-35}.$$

2. The linear regression model takes the form

$$\hat{P}_L(N) = 65.215196 - 0.000731458N, \quad R_L^2 = 0.691114.$$

3. The power-law (Heaps) regression model takes the form

$$\hat{P}_H(N) = 269.061418N^{-0.195427}, \quad R_H^2 = 0.941920, \quad R_{\log}^2 = 0.945982.$$

4. The power-law model's error, $\text{MSE}_H = 9.158715$, is markedly smaller than the linear model's error, $\text{MSE}_L = 48.708482$, and this advantage is confirmed by an AIC/BIC comparison computed on a common scale (Table 3);

5. The absolute number of unique words obeys the Heaps-type sublinear growth law

$$V(N) = 2.69061418N^{0.804573}.$$

which was proved (Statement 1) to be strictly increasing, strictly concave, and asymptotically sublinear relative to N , while the corresponding percentage share $P(N)$ is strictly decreasing and tends to zero as $N \rightarrow \infty$.

Thus, for this corpus, expressing the relationship between total word count and the percentage share of unique words by means of a power-law model is preferable to a linear regression, both statistically and asymptotically, and the Heaps-law hypothesis tested in this study is confirmed as the substantially better-supported of

the two candidate methods. Beyond its purely descriptive value, the resulting closed-form vocabulary-growth law directly answers the motivating question of this study: because vocabulary size grows sublinearly rather than proportionally with text length, TF-IDF-based pipelines for Uzbek-language text similarity, summarisation, classification, and clustering do not incur a severe curse-of-dimensionality penalty at the book scale at which these tasks are typically applied, and the resulting vector dimensionality and memory requirements can be forecast in closed form (Section 3.5) before a corpus is even fully processed. Future work should extend this analysis to lemma-level and morpheme-level vocabulary counts, to additional Uzbek text genres (in particular web and social-media text, administrative and legal documents, and spoken transcripts), and to a within-text (single-document) formulation of Heaps' law for comparison with the multi-document exponent estimated here; a further natural extension is to estimate directly the Zipfian word-frequency exponent for the same corpus and to test empirically whether it matches the value $z \approx 1.243$ implied by the estimated Heaps exponent via the established theoretical correspondence between the two laws [7–9]. A further practically oriented extension would be to validate the memory-forecasting methodology of Section 3.5 directly against the actual vocabulary sizes and memory consumption observed when building the TF-IDF-based similarity, summarisation, classification, and clustering pipelines that motivated this study (e.g., [24–26]) at progressively larger corpus scales, thereby empirically calibrating not just the statistical model but also the practical accuracy of the downstream engineering forecasts it enables. Taken together, the statistical, mathematical, and applied strands of this study support a single unifying conclusion: for the Uzbek language, as for the many other languages in which Heaps' law has been documented, the growth of vocabulary with text length is not a matter of chance but a regular, quantifiable, and practically exploitable property of the language's written form, and making that property explicit – in the form of a validated regression equation rather than an informal impression – is what allows the curse of dimensionality to be anticipated and, at realistic book scale, shown not to be a

serious obstacle for TF-IDF-based Uzbek-language natural-language-processing systems.

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UDC: 621.891:678.046:622.73

ENHANCING THE WEAR RESISTANCE OF THE WORKING SURFACES OF MILL AND PUMP COMPONENTS USING AN ULTRA-HIGH-MOLECULAR-WEIGHT POLYMER COMPOSITE: A COMBINED TRIBOLOGICAL, MATHEMATICAL, AND INDUSTRIAL STUDY

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Annotatsiya. Tegirmonlar va suspenziyalı nasoslarning ishchi elementlari konchilik, metallurgiya va mineral xomashyoni qayta ishlash jarayonlarida abraziv yeyilish, eroziya va kavitatsiya taʼsirida shikastlanadi. Mazkur tadqiqotda oʻta yuqori molekulyar massali polietilen (UHMWPE) asosida grafit va qisqa tolalar bilan mustahkamlangan UVKP polimer kompozit materiali ishlab chiqildi. Material 30–160 °C haroratda va 250–300 MPa bosim ostida presslash orqali olindi. Uning tribologik, mexanik va fizik xususiyatlari eksperimental usullar yordamida baholandi. Sanoat sinovlari natijasida detallar xizmat muddatining 3 baravargacha oshishi, montaj uchun toʻxtash vaqtining 40% ga va elektr energiyasi sarfining 2–25% ga kamayishi qayd etildi. Olingan natijalar UVKP kompozitining

ogʻir sanoat sharoitlarida yeyilishga chidamlilikni oshirish va ekspluatatsiya xarajatlarini kamaytirish imkoniyatlarini koʻrsatadi.

Kalit soʻzlar: *UVKP, polimer kompozit, yeyilishga chidamlilik, UHMWPE, tegirmon, nasos, abraziv yeyilish, energiya samaradorligi.*

Аннотация. Рабочие элементы мельниц и шламовых насосов в горнодобывающей, металлургической промышленности и при переработке минерального сырья подвергаются абразивному износу, эрозии и кавитации. В исследовании разработан полимерный композиционный материал UVKP на основе сверхвысокомолекулярного полиэтилена (UHMWPE), армированного графитом и короткими волокнами. Материал получен методом прессования при температуре 130–160 °C и давлении 250–300 МПа. Трибологические, механические и физические свойства оценивались экспериментальными методами. По результатам промышленных испытаний срок службы деталей увеличился до трёх раз, время простоев при монтаже сократилось на 40%, а энергопотребление — на 12–25%. Полученные результаты подтверждают перспективность применения композита UVKP для повышения износостойкости оборудования и снижения эксплуатационных затрат в тяжёлых промышленных условиях.

Ключевые слова: *UVKP, полимерный композит, износостойкость, UHMWPE, мельница, насос, абразивный износ, энергоэффективность.*

Abstract. Working components of mills and slurry pumps in mining, metallurgy, and mineral processing are exposed to abrasive wear, erosion, and cavitation. This study presents a UVKP polymer composite based on ultra-high-molecular-weight polyethylene (UHMWPE), reinforced with graphite and short fibers. The material was produced by compression molding at 130–160 °C and 250–300 MPa. Its tribological, mechanical, and physical properties were evaluated using experimental methods. Industrial trials demonstrated up to a threefold increase in component service life, a 40% reduction in installation downtime, and a 12–25% decrease in energy consumption. These findings indicate that UVKP composites offer a promising solution for improving wear resistance and reducing operating costs in demanding industrial environments.

Key words: *UVKP, polymer composite, wear resistance, UHMWPE, mill, pump, abrasive wear, energy efficiency.*

Introduction

The extraction, comminution, and hydro-transport of mineral resources in large-scale extractive metallurgy and beneficiation plants require processing hundreds of millions of tons of polyphase ores annually [1-3]. Critical mechanical unit operations—predominantly autogenous (AG), semi-autogenous grinding (SAG), and

secondary/tertiary ball mills, alongside high-capacity centrifugal slurry pumps—operate under exceptionally aggressive tribological, dynamic, and chemical conditions [4-7]. Working surfaces of these installations, such as mill shell liners, lifter bars, discharge grates, slurry pump volutes, impellers, and suction throat bushes, are continuously subjected to intensive multi-component surface wear [8-11, 13, 14].



Figure 1. Severe abrasive wear, micro-trough, and local cavitation wear in the metal working parts of the slurry pump observed in industrial operation [4, 5, 13, 14].

In commercial milling circuits, mineral pulps consist of dense aqueous suspensions of crushed ore containing quartz (SiO_2 , Mohs hardness 7.0, Vickers microhardness $\sim 1100\text{--}1200$ HV), pyrite (FeS_2 , Mohs hardness 6.0–6.5), chalcopyrite, magnetite, and corundum [12-15]. Pulp solid concentrations typically vary between 50% and 75% by mass, with slurry flow velocities in pump manifolds exceeding 10 to 20 m/s. Under these violent fluid-solid flow dynamics, component surfaces are continuously scoured by billions of sharp, angular mineral micro-asperities while simultaneously sustaining severe cyclic shock impacts from cascading and cataracting forged steel grinding balls (diameters 80–125 mm, unit masses 2.1–8.0 kg) possessing kinetic impact energies ranging from 150 to 550 Joules per strike [19-20]. This harsh combined loading accelerates surface spallation, catastrophic through-wall fracture, and severe operational disruptions.

Experimental Materials, Synthesis Protocols, and Characterization

The polymer matrix comprised medical- and industrial-grade UHMWPE powder (GUR 4120, Celanese Corp., Germany; $M_w \approx 4.2 \times 10^6$ g/mol, $d_{50} = 110\text{--}130$ μm , solid density 0.930 g/cm³, MFI ≈ 0 g/10 min at 190°C / 21.6 kg) [15]. Synthetic micrographite powder (Asbury Carbons, USA; purity ≥ 99.5 wt.%, $d_{50} < 45$ μm , specific surface area $\text{SSA} \approx 8.5$ m²/g, ash content < 0.1 wt.%) was incorporated at 15 wt.% to provide self-lubrication and enhance thermal conductivity [12, 55]. Short crushed mineral/carbon fibers (filament diameter 7.0 ± 0.5 μm , length distribution 200–300 μm , aspect ratio $L/D \approx 35$) were added at 5 wt.% to increase compressive modulus and arrest microcracks [9, 10]. A synergistic antioxidant blend (Irganox 1010 / Irgafos 168, 0.5 wt.%, BASF SE) was added to protect molecular chains during thermal processing [16].

Table 1. Detailed formulation and chemical-morphological specifications of UVKP composite constituents.

Constituent Component	Functional Role	Physicochemical Specifications	Formulation Fraction
UHMWPE Matrix (GUR 4120)	Base thermoplastic resin	$M_w \approx 4.2 \times 10^6$ g/mol, $d_{50} = 110\text{-}130$ μm , Density = 0.93 g/cm ³	79.5 wt.%
Synthetic Micrographite	Solid lubricant / Thermal conductor	Purity $\geq 99.5\%$, $d_{50} < 45$ μm , SSA = 8.5 m ² /g, Ash < 0.1%	15.0 wt.%
Short Mineral/Carbon Fibers	Structural reinforcement / Crack arrestor	Diameter = 7.0 μm , Length = 200-300 μm , Aspect Ratio L/D ≈ 35	5.0 wt.%
Thermo-Oxidative Stabilizer	Radical scavenger / Processing stabilizer	Irganox 1010 / Irgafos 168 (1:1 synergist blend)	0.5 wt.%

Mathematical Modeling and Theoretical Tribo-Physics

In industrial tumbling mills, three-body abrasive wear occurs as angular mineral grains roll and slide between the liner and grinding balls [8-17]. Total penetration depth (h_t) comprises elastic (h_e) and plastic/ploughing (h_p) components: $h_t = h_e + h_p$ [21, 30]. Irreversible volumetric loss (ΔV_p) per unit sliding distance is expressed through the contact mechanics formulation [29, 30]:

$$\Delta V_p = \phi_c \cdot \Sigma \left(\frac{F_{\{n,i\}}}{H_{\{eff\}}} \right) \cdot \tan(\theta) \cdot \left[1 - \psi \left(\frac{H}{E} \right) \right] \quad (3)$$

where N_a is the number of active abrasive asperities, ϕ_c is cutting transition probability ($\phi_c \approx 0.05\text{--}0.12$ for UVKP vs $0.65\text{--}0.85$ for 45Kh steel) [30], and $\psi(H/E)$ is a non-linear viscoelastic recovery function [11, 18]:

$$\psi \left(\frac{H}{E} \right) = \kappa_0 \cdot \left(\frac{H}{E} \right)^{\{\alpha_m\}} \quad (4)$$

For structural steel (45Kh), high stiffness ($E \approx 210$ GPa) yields a low elastic resilience ratio ($H/E \approx 0.024$), causing severe plastic cutting [21, 23]. In contrast, the UVKP composite combines moderate hardness ($H \approx 95$ MPa) with compliant elasticity ($E \approx 1.25$ GPa), providing $H/E \approx 0.076$ [8, 11]. This higher resilience allows the composite to absorb kinetic impacts elastically and recover without microcrack propagation, suppressing micro-cutting by over 75% [11, 20].

Results and Discussion

Consolidated UVKP composites underwent extensive mechanical characterization. Table 4 compiles comparative performance data against standard industrial metallic alloys [8-19].

Table 4. Comprehensive physicochemical, mechanical, and tribological properties of UVKP composite versus standard steels.

Physical / Mechanical Property	UVKP Composite	45Kh Structural Steel	12Kh18N 10T Stainless	Hadfield Steel (110G13 L)	Units	Comparative Gain
Density (ρ)	1.30	7.80	7.90	7.85	g/cm ³	-83.3% vs steel
Compressive Yield Strength (σ_y)	112 $\pm$ 4	620 $\pm$ 15	540 $\pm$ 12	450 $\pm$ 20	MPa	Sufficient for mill loads
Flexural Strength (σ_f)	88 $\pm$ 3	850 $\pm$ 25	720 $\pm$ 20	800 $\pm$ 30	MPa	High toughness
Elastic Modulus (E)	1.25 $\pm$ 0.08	210 $\pm$ 5	195 $\pm$ 5	200 $\pm$ 5	GPa	Compliant viscoelastic
Shore D Hardness / HB	78 $\pm$ 2 (Shore D)	210 (HB)	180 (HB)	220-550 (HB)	-	Balanced hardness
Elastic Resilience Ratio (H/E)	0.076	0.024	0.021	0.023	-	+216% vs steel
Friction Coefficient (μ , dry vs steel)	0.08 - 0.14	0.45 - 0.65	0.50 - 0.70	0.40 - 0.60	-	3.5x to 5x reduction
Specific Wear Rate (I_w)	0.028 $\pm$ 0.004	0.185 $\pm$ 0.02	0.210 $\pm$ 0.03	0.145 $\pm$ 0.02	mm ³ /(N·m)	5.2x higher resistance
Thermal Conductivity (κ)	1.48 $\pm$ 0.05	46.0 $\pm$ 1.2	16.0 $\pm$ 0.8	14.0 $\pm$ 0.6	W/(m·K)	Enhanced via graphite
Continuous Service Temperature	-50 to +110	-40 to +450	-196 to +600	-40 to +350	°C	Full mining range

Conclusion and Recommendations

1. Severe Multi-Mechanism Wear Mitigation and Industrial Pump Protection: Functional elements of grinding mills and slurry pumps undergo accelerated degradation driven by simultaneous three-body abrasive micro-cutting, cyclic dynamic impact fatigue, and aggressive cavitation micro-jet peeling coupled with electrochemical dissolution (as evidenced by Image 1). The newly developed UVKP composite (UHMWPE matrix reinforced with 15 wt.% synthetic micrographite and 5 wt.% short mineral/carbon fibers) successfully eliminates brittle failure and galvanic corrosion ($W_{corr} \approx 0$ in Eq. 1).

2. Solid-State High-Pressure Consolidation Protocol: Implementing the optimized multi-stage compression molding process (compaction pressure $P = 250\text{--}300$ MPa, isothermal sintering dwell at $T = 130\text{--}160$ °C, and programmed annealed cooling at $V_c = 0.8\text{--}1.2$ °C/min, Table 2 and Table 3) achieved complete macromolecular chain reptation across grain boundaries, eliminating internal microporosity and residual thermal macro-stresses.

3. High Elastic Resilience and Cutting Suppression: The UVKP composite demonstrates an optimal mechanical combination of high elastic resilience (ratio $H/E \approx 0.076$), compressive yield strength of $\sigma_y = 112 \pm 4$ MPa, low stable friction coefficient ($\mu = 0.08\text{--}0.14$), and an exceptionally low specific wear rate ($I_w = 0.02\text{--}0.05$ mm³/(N·m), Table 1 and Table 4). As derived in the contact mechanics formulations (Eqs. 3 and 4), the high H/E resilience ratio allows the material to absorb kinetic asperity indentation elastically, reducing abrasive cutting by over 75% compared to structural steel (45Kh).

4. Self-Lubricating Transfer Film Kinetics and Interfacial Heat Dissipation: Mathematical modeling and tribological observations (Figure 1 and Figure 2) verified that interfacial shear stresses induce the dynamic formation of a coherent graphite-polymeric transfer film (equilibrium thickness $h_{tf} \approx 1.5\text{--}3.0$ μm, Eqs. 12 and 13). Concurrently, the enhanced effective thermal conductivity ($\kappa_c = 1.48$ W/(m·K), Eq. 7) lowers localized flash contact temperatures (T_{flash} , Eqs. 5 and 6) by 42–55 °C compared to unfilled UHMWPE, preventing thermal softening.

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Section 3. ACTUAL PROBLEMS OF HISTORY, PHILOSOPHY AND SOCIOLOGY

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STAGES OF HISTORICAL DEVELOPMENT OF THE PUBLIC EDUCATION SYSTEM IN UZBEKISTAN

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Annotatsiya. Mazkur maqolada O‘zbekiston Respublikasi xalq ta’limi tizimining tarixiy rivojlanish bosqichlari, ayniqsa, mustaqillikdan keyingi davrda amalga oshirilgan islohotlar tizimli ravishda tahlil qilinadi. Tadqiqotda O‘zbekiston hududida ta’lim va ma’rifatning qadimgi davrlardan shakllanishi, islom uyg‘onish davri, temuriylar davri, jadidchilik harakati hamda sovet davridagi ta’lim tizimining o‘ziga xos xususiyatlari yoritiladi. Asosiy e’tibor 1991–2026-yillarda milliy ta’lim tizimining shakllanishi va modernizatsiyasi, ta’limning huquqiy asoslarini takomillashtirish, “Kadrlar tayyorlash milliy dasturi”ni amalga oshirish, uzluksiz ta’lim tamoyilining rivojlanishi, 2017-yildan keyingi keng ko‘lamli islohotlar, ta’lim sifatini oshirish, pedagog maqomini yuksaltirish, xalqaro ta’lim standartlarini joriy etish va raqamlashtirish jarayonlariga qaratiladi. Tadqiqot natijalari O‘zbekistonda ta’lim tizimini modernizatsiya qilish mustaqillik yillarida davlat siyosatining strategik ustuvor yo‘nalishlaridan biriga aylanganini ko‘rsatadi.

Kalit so‘zlar: *ta’lim tizimi, xalq ta’limi, tarixiy rivojlanish, mustaqillik, ta’lim islohotlari, modernizatsiya, uzluksiz ta’lim, pedagogika, raqamlashtirish, ta’lim sifati, xalqaro standartlar.*

Аннотация. В статье системно анализируются исторические этапы развития системы народного образования Республики Узбекистан, особенно реформы, осуществлённые в период после обретения независимости. Рассматриваются исторические особенности становления образования и просвещения на территории Узбекистана с древнейших времён, период исламского культурного подъёма, эпоха Тимуридов, джадидское движение и советская система образования. Основное внимание уделяется

формированию и модернизации национальной системы образования в 1991–2026 годах, совершенствованию её правовой базы, реализации Национальной программы подготовки кадров, развитию принципа непрерывного образования, широкомасштабным реформам после 2017 года, повышению качества образования, укреплению статуса педагогов, внедрению международных образовательных стандартов и цифровизации. Результаты исследования показывают, что модернизация системы образования стала одним из стратегических приоритетов государственной политики Узбекистана в годы независимости.

***Ключевые слова:** система образования, народное образование, историческое развитие, независимость, образовательные реформы, модернизация, непрерывное образование, педагогика, цифровизация, качество образования, международные стандарты.*

Abstract. This article systematically analyzes the historical stages of development of the public education system of the Republic of Uzbekistan, with particular attention to the reforms implemented after independence. The study examines the historical development of education and enlightenment in the territory of Uzbekistan from ancient times, including the period of Islamic cultural advancement, the Timurid era, the Jadid movement, and the Soviet education system. Particular attention is devoted to the formation and modernization of the national education system during 1991–2026, the improvement of its legal framework, implementation of the National Program for Personnel Training, development of lifelong learning, large-scale reforms after 2017, improvement of educational quality, strengthening the professional status of teachers, introduction of international educational standards, and digital transformation. The findings demonstrate that modernization of the education system has become one of the strategic priorities of state policy in Uzbekistan during the years of independence.

Key words: education system, public education, historical development, independence, educational reforms, modernization, lifelong learning, pedagogy, digitalization, education quality, international standards.

Introduction

Education is one of the main institutions ensuring the socio-economic, political and spiritual development of society. In modern conditions, the competitiveness of states is increasingly determined by the quality of human capital, the knowledge and skills of the population, the scientific and intellectual potential, and the effectiveness of the education system. After the Republic of Uzbekistan gained independence, the reorganization of the education system based on national interests, historical and cultural values, and modern international requirements became one of the important directions of state policy. In the early years of independence, much attention was paid to creating the legal and organizational foundations of the national education system, updating the content of education, and forming a system for training qualified personnel. The “National Program for Personnel Training” adopted in 1997 began an important stage in the comprehensive reform of the education system in Uzbekistan. This program set the tasks of forming a system of continuous education, strengthening ties between education and production, increasing the professional potential of pedagogical personnel, and training competitive specialists. At the next stage, the legal framework of the education system was further improved. The adoption of the new edition of the Law “On Education” in 2020 served to regulate the continuity, forms and content of education based on modern requirements. The reforms implemented in the education sector since 2017 have brought the system to a new level. The processes of expanding the scope of preschool education, improving the quality of general secondary education, supporting teachers, creating new generation textbooks, participating in international assessment programs, and introducing digital technologies into the educational process have accelerated. Therefore, studying the stages of historical development of the public education system of Uzbekistan is important not only for understanding historical processes, but also for scientifically assessing the content and prospects of today's education policy.

Research Methodology

The research used methods of historicity, systematics, comparative-historical analysis, retrospective analysis, study of regulatory legal documents and analysis of

statistical data. The source basis of the research was the laws of the Republic of Uzbekistan in the field of education, presidential decrees and resolutions, state programs, data of the National Statistics Committee, relevant materials of UNESCO and the World Bank, as well as scientific literature on the history of education. In the periodization of historical processes, the content of the education system, management mechanisms, legal frameworks, institutional structure and changes in the quality of education were taken as the main criteria.

The development of education in Uzbekistan in ancient and medieval times. Education and upbringing processes in Uzbekistan date back to ancient times. The initial forms of education were associated with family, community and labor activity. The main forms of upbringing were the transmission of life experience, professional skills, traditions and moral values to the younger generation. Starting from the 7th-8th centuries, as a result of the widespread spread of Islamic culture in Central Asia, the activities of schools and madrasas developed. While reading, writing, the basics of the Quran and Arabic were taught in schools, in madrasas, along with religious knowledge, such subjects as logic, philosophy, mathematics, astronomy and medicine were also studied. In the 9th-12th centuries, the rise of science and culture in the regions of Transoxiana and Khorezm had a great influence on the development of education. The scientific heritage of Muhammad al-Khwarizmi, Ahmad al-Farghani, Abu Rayhan Beruni, Abu Ali ibn Sino and other scholars made a great contribution to the development of world science. The Khorezm Mamun Academy was also one of the important centers of this scientific and educational upsurge. The experience of this period shows that education in Uzbekistan developed inextricably linked with the development of science.

Education and science during the Timurid era. In the 14th–15th centuries, during the Timurid period, Samarkand and Herat became major centers of science and culture. As a result of the development of educational institutions, madrasahs, and scientific centers, mathematics, astronomy, history, philosophy, literature, and other sciences flourished. Mirzo Ulugbek's establishment of a madrasah and observatory in Samarkand was a vivid example of the development of scientific education. During this period, the

scientific orientation of the educational process intensified, and special attention was paid to the exact sciences. The educational experience of the Timurid period demonstrated the importance of scientific thinking, the tradition of a teacher-student relationship, and the scientific environment in the development of society.

The Jadid movement and the modernization of education. The Jadid movement in the late 19th and early 20th centuries marked a significant turning point in the history of education in Uzbekistan. Jadid enlighteners promoted modern knowledge and enlightenment as an important condition for the development of society. Enlightenment figures such as Mahmudkhodja Behbudiy, Munavvarqori Abdurashidkhanov, Abdulla Avloniy, and Abdurauf Fitrat made a significant contribution to the organization of new-style schools, the creation of textbooks, the teaching of secular sciences, and the improvement of educational methodology. The issues of national consciousness, patriotism, the pursuit of science, and the formation of a modern worldview occupied a central place in the Jadids' views on education. Therefore, the Jadid movement can be considered an important stage in the history of the modernization of the education system of Uzbekistan.

During the Soviet period, large-scale work was carried out in Uzbekistan to increase general literacy, expand the school network, and increase the coverage of the population with education. During this period, centralized management of the education system strengthened. The popularization of general education and the expansion of the school network contributed to an increase in the level of literacy. However, the content of education was closely related to Soviet ideology, and a significant part of the educational process was formed on the basis of centralized political and ideological requirements. Thus, the Soviet period in the history of public education in Uzbekistan is characterized not only by the formation of mass education, but also by the ideological orientation of education.

Formation of the legal framework of the national education system in 1991–1997. Uzbekistan's achievement of state independence in 1991 marked a new historical stage in the development of the education system. In the early years of independence, the formation of the national content of education, the return of historical and spiritual

heritage to the educational process, and the creation of a national personnel training system were among the important tasks. In order to strengthen the legal foundations of education, relevant laws and state programs were adopted. The process of adapting the education system to the needs of an independent state, revising curricula, and creating new textbooks began. This period can be characterized as a stage in which the institutional foundations of national education policy were formed.

Development of the education system based on the "National Personnel Training Program" in 1997–2016. The adoption of the Law “On Education” and the “National Program for Personnel Training” in 1997 marked a new stage in the development of the education system of Uzbekistan. The national program was aimed at developing education based on the interests of the individual, society and the state, forming a system of continuous education, strengthening cooperation between education and production, and training competitive personnel. During this period, attention was paid to ensuring the continuity of general secondary education, secondary specialized, vocational education and higher education. Also, the material and technical base of educational institutions was strengthened, new educational buildings were built, and the possibilities for introducing modern pedagogical technologies into the educational process expanded.

New reforms in education in 2017–2021. Since 2017, large-scale institutional and substantive reforms have been implemented in the education system in Uzbekistan. The development of preschool education as a separate system, improving the quality of general secondary education, supporting teachers, and expanding the scope of higher education have become important areas. New approaches have been introduced in educational management, and great attention has been paid to strengthening the material and technical base of educational institutions, creating modern textbooks, and improving the qualifications of teachers. During this period, attention has also increased to the issue of assessing education based on international standards. Approaches aimed at assessing students' knowledge not only theoretically, but also based on practical competencies began to develop.

Legal and institutional renewal of the education system in 2020–2022. On September 23, 2020, the Law of the Republic of Uzbekistan “On Education” in a new

edition was adopted. This law established the modern legal framework of the education system. The law legally strengthened issues such as the expansion of forms of education, continuity of education, distance learning, inclusive education, and equal opportunities for education. In 2022 The National Program for the Development of Public Education for 2022–2026 was approved. It set out tasks such as the full implementation of the National Curriculum, the creation of modern textbooks, increasing the prestige of the teaching profession, ensuring continuous professional development of teachers, and the introduction of international educational programs.

Modernization of the education system in 2023–2026. The development of the education system in 2023–2026 continued in connection with the strategic goals of "Uzbekistan - 2030". The strategy includes the expansion of preschool education coverage by 2030.

The goal is to reach 80%, and the current indicator for 2025 is 78%. According to official data from early 2026, preschool education coverage has reached 78%. At the same time, new measures have been set to reduce disparities between regions, improve the skills of teachers, and improve the school preparation system. It is also planned to establish state preschool educational organizations "New Generation" in 2026-2028

Main trends in the development of the education system. An analysis of the development of the public education system in Uzbekistan after independence allows us to identify the following main trends:

The first trend is to strengthen the national foundations of education. After independence, the integration of national history, culture, spirituality, and values into educational content increased.

The second trend is the development of a continuing education system. There has been increased attention to ensuring the continuity of education from pre-school to higher education and professional development.

The third trend is assessing the quality of education based on international criteria. The importance of international assessment programs and competency-based approaches in the Uzbek education system is increasing.

The fourth trend is digitalization. E-learning resources, digital platforms, and distance learning opportunities are becoming an important part of the educational process.

The fifth trend is to raise the status of teachers. The professional development, promotion of teachers' work, and improvement of their social status have become one of the important areas of state policy.

The sixth trend is the shift to STEM and competency-based education. In modern education, great attention is paid not only to the acquisition of knowledge by the student, but also to the development of practical application of knowledge, problem-solving, critical thinking, and creativity skills.

Results and Discussion

Historical analysis shows that the education system of Uzbekistan has gone through several important stages of transformation. While in the pre-independence period the education system was under the strong influence of state and ideological control, in the post-independence period the creation of a national education model became a priority. While 1991–1997 was the period of formation of the legal framework of the national education system, 1997–2016 is characterized by the development of a system of continuous education and personnel training. At the same time, in the future development of the education system, such tasks as reducing the gap in the quality of education between regions, improving the skills of teachers, ensuring equal access to educational resources, strengthening the functional literacy of students, and the effective use of digital technologies remain relevant.

Conclusion and Recommendations

The historical development of the public education system of Uzbekistan is a continuous continuation of the centuries-old scientific and educational heritage and educational experiences formed in different historical periods. Forms of education in ancient times, schools and madrasas during the era of Islamic culture, scientific and educational upsurge during the Timurid era, the ideas of modernization of education of the Jadids, and the experience of mass education during the Soviet era constitute the historical roots of today's education system. The 2020 Law "On Education" strengthened

the modern legal framework of the education system. The National Program for 2022–2026 is aimed at updating the content of school education, ensuring the professional development of teachers, and introducing international educational programs. The further development of the education system in 2026 is associated with increasing the coverage of preschool education, improving the quality of education, strengthening the potential of pedagogical personnel, developing STEM and engineering education, and the widespread use of digital technologies. The state policy for 2026 also sets the goal of increasing preschool education coverage to 80%. Thus, the historical development of the public education system of Uzbekistan is not a simple set of institutional changes, but a complex transformational process inextricably linked to the socio-economic and spiritual development of society. Its current stage is based on the integration of international experience, digital technologies, and competency-based approaches while preserving the national pedagogical heritage.

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Section 4. ACTUAL PROBLEMS IN MODERN AGRICULTURE

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THE EFFECT OF SEASONAL VARIATIONS, TYPE AND DEGREE OF SALINITY ON THE ACTIVITY OF CELLULOSE-DEGRADING MICROORGANISMS IN IRRIGATED SEROZEM-MEADOW SOILS OF THE ARID ZONE

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Annotatsiya. Tadqiqotda sug'oriladigan sur tusli qo'ng'ir-o'tloqi tuproqlarda sho'rlanish darajasi, tuproq qatlamining chuqurligi va mavsumiy sharoitlarning sellyuloza parchalovchi mikroorganizmlar soniga ta'siri o'rganildi. Natijalarga ko'ra, sho'rlanish kuchayishi va tuproq profilining chuqurlashishi bilan mikroorganizmlar soni kamaygan, eng yuqori ko'rsatkichlar esa sho'rlanmagan tuproqlarning 0-25 sm qatlamida, ayniqsa yoz mavsumida qayd etilgan.

Kalit so'zlar. sug'oriladigan tuproqlar, sho'rlanish, sellyuloza parchalovchi mikroorganizmlar, tuproq profili, mavsumiy dinamika, mikrobiologik faollik, organik modda.

Аннотация. Изучено влияние степени засоления, глубины почвенного профиля и сезонных условий на численность целлюлозоразлагающих микроорганизмов в орошаемых серозёмно-луговых почвах. Установлено, что с усилением засоления и увеличением глубины почвы численность микроорганизмов снижается, а наиболее высокие показатели отмечаются в незасоленном слое 0-25 см, особенно в летний период.

Ключевые слова. Орошаемые почвы, засоление, целлюлозоразлагающие микроорганизмы, почвенный профиль, сезонная динамика, микробиологическая активность, органическое вещество.

Abstract. The study evaluated the effects of soil salinity, profile depth, and seasonal conditions on the abundance of cellulose-decomposing microorganisms in irrigated sierozem-meadow soils. The results showed that microbial abundance

decreased with increasing salinity and soil depth, while the highest values were recorded in the non-saline 0-25 cm layer, particularly during summer.

Key words. *irrigated soils, salinity, cellulose-decomposing microorganisms, soil profile, seasonal dynamics, microbiological activity, organic matter.*

Introduction

Arid regions are characterized by limited water resources, high evaporation rates, sharp temperature fluctuations, and widespread soil salinization. Under irrigated farming conditions, these factors significantly influence not only the soil's physical-hydrological and agrochemical properties but also the state of its biological component. Soil microflora, which plays a role in the transformation of organic matter and the cycling of major nutrients, is particularly sensitive to changes in environmental conditions [1,2].

Cellulose-degrading microorganisms play an important role in the structure of the soil microbial community. They break down plant residues containing cellulose and thereby contribute to the mineralization of organic matter, the formation of humic compounds, and the mobilization of nutrients. The intensity of cellulolytic processes can be considered an indicator of soil biological activity and the degree to which conditions are favorable for microbiological processes [3,4].

In irrigated serozem-meadow soils of the arid zone, the activity of cellulose-degrading microorganisms is determined by the combined effect of a number of factors, among which the degree and type of salinization, moisture content, temperature, organic matter content, particle size distribution, and soil horizon depth are of particular importance. Elevated concentrations of readily soluble salts increase the osmotic pressure of the soil solution, reduce moisture availability to microorganisms, and can disrupt their physiological and biochemical processes. Consequently, as salinization increases, a decrease in the abundance and functional activity of a significant portion of the soil microbiota is typically observed [5,6].

In addition to salinity, seasonal changes in hydrothermal conditions have a significant impact on cellulolytic activity. In spring, summer, and fall, the ratio of soil temperature to moisture content changes significantly, which affects the intensity of

microbiological processes. Under favorable temperature conditions with sufficient moisture, the decomposition of plant residues accelerates, whereas moisture deficiency, soil overheating, or salt accumulation can sharply limit microbial activity. Therefore, seasonal fluctuations in the abundance of cellulose-degrading microflora serve as an important indicator of changes in the ecological condition of irrigated soils [7,8].

For arid regions, this issue is particularly relevant, since prolonged irrigation in areas with inadequate drainage can lead to secondary salinization, the redistribution of salts throughout the soil profile, and a deterioration in the soil's biological condition. At the same time, serozem-meadow soils, which have a relatively low organic matter content and limited biological activity, may react particularly acutely to an increase in salt load [9,10].

Studying the seasonal dynamics of cellulose-degrading microorganisms as a function of the type and degree of salinization allows us to assess the direction of changes in soil biological activity, identify the most vulnerable horizons, and determine the extent to which salt stress affects organic matter transformation processes. Such data are crucial for providing a scientific basis for measures aimed at preserving and restoring the fertility of irrigated soils in arid regions.

The objective is to assess the impact of seasonal dynamics, type, and degree of salinization on the activity and abundance of cellulose-degrading microorganisms in irrigated serozem-meadow soils of the arid zone, as well as to establish patterns of their variation across the seasons and along the soil profile.

Materials and Methods

The studies were conducted in an arid zone within the Korovulbazar District of the Bukhara Region, on irrigated serozem-meadow soils. This area was selected as the study site due to the widespread occurrence of saline irrigated soils, seasonal variability in hydrothermal conditions, and the need to assess the impact of salinization on soil microbiological activity.

Field studies were conducted on plots with varying degrees of salinization by establishing soil profiles and collecting samples. Soil samples were collected from

designated layers of the soil profile and transported to the laboratory for microbiological analysis.

To prepare a soil suspension, 10 g of soil was taken from each sample and placed in a container containing 90 mL of sterile distilled water. The resulting mixture was shaken thoroughly for 5 minutes, after which the first dilution was obtained. Next, 10 mL of the suspension from the first dilution was drawn up with a sterile pipette and transferred to another container containing 90 mL of sterile distilled water, yielding the second dilution. This process of sequential dilutions was continued up to the fourth dilution. To determine the population size of cellulose-degrading microorganisms belonging to one of the physiological groups of the soil microflora, a fourth-dilution soil suspension was used. The suspension was inoculated onto a culture medium that supports the growth of microorganisms capable of degrading cellulose. After incubation, the number of colonies formed was counted, and the population of cellulose-degrading microorganisms was expressed in thousands of colony-forming units per 1 g of soil (thousands of CFU/g of soil).

The studies were conducted in the spring, summer, and fall, which made it possible to track the seasonal dynamics of the population of cellulose-degrading microorganisms. The analysis also took into account soil depth, soil type, and the degree of salinization. For comparative evaluation, unsalinized, slightly saline, and moderately saline soil variants were identified.

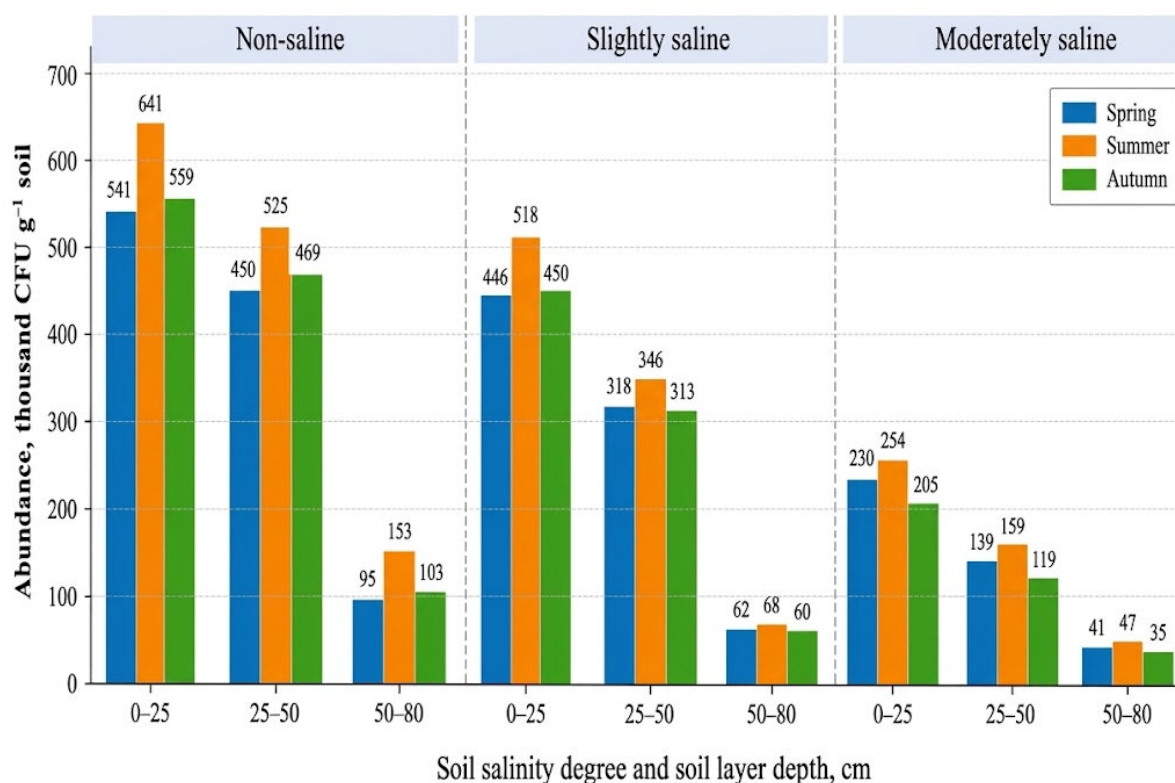
Results and Discussion

The results obtained indicate a significant influence of salinity level, soil profile depth, and seasonal conditions on the abundance of cellulose-degrading microorganisms in irrigated serozem-meadow soils. A general pattern is observed across all studied variants: as salinity increases and the depth of the soil layer deepens, the abundance of this physiological group of microorganisms gradually decreases. At the same time, the highest values are observed in most cases during the summer. In unsalted soils, the maximum population of cellulose-degrading microorganisms was found in the 0-25 cm layer. In the spring, it was 541,000 CFU/g of soil; in the summer, it increased to 641,000; and in the fall, it decreased to 559,000. In the 25-50 cm layer, the corresponding values

were 450, 525, and 469 thousand, whereas at a depth of 50-80 cm, they decreased to 95-153 thousand. This indicates that cellulose-degrading microflora is predominantly localized in the upper part of the soil profile.

A similar trend was observed in slightly saline soils, although the absolute values were lower. In the 0-25 cm layer, the microorganism count was 446,000 in the spring, 518,000 in the summer, and 450,000 in the fall. At a depth of 25-50 cm, the counts decreased to 318,000-346,000, and in the 50-80 cm layer, to 60,000-68,000. Consequently, even mild salinization has a noticeable limiting effect on the development of cellulose-degrading microflora.

The most pronounced decrease in abundance was observed in moderately saline soils. In the upper horizon, the counts were 230,000 in the spring, 254,000 in the summer, and 205,000 in the fall. In the 25-50 cm layer, they decreased to 139,000-159,000, and in the 50-80 cm horizon to 35,000-47,000. Compared to the non-salinized treatments, the microorganism population in the upper layer of moderately saline soil was more than two times lower (Fig. 1).



Note: The salinity type in all treatments is C.

Figure 1. Seasonal trends in the abundance of cellulose-degrading microorganisms in irrigated serozem-meadow soils as a function of salinity level and soil depth

Seasonal dynamics were characterized by a predominant increase in population size during the summer. This pattern was most clearly evident in the unsalinated and slightly saline treatments. This may be associated with more favorable temperature conditions, increased plant root exudation, and an intensive influx of available organic compounds into the soil. In the fall, a slight decrease in these indicators was observed, which was likely due to changes in temperature and moisture conditions and a decline in the activity of microbiological processes. As the depth of the soil profile increased, the abundance of cellulose-degrading microorganisms decreased systematically, regardless of the degree of salinity. This pattern can be explained by a decrease in organic matter content, a reduction in the amount of plant residues and root exudates, as well as less favorable aeration and water regime conditions in the lower horizons. Overall, the data indicate that the degree of salinity is a significant ecological factor limiting the growth of cellulose-degrading microorganisms. An increase in the concentration of soluble salts contributes to heightened osmotic stress and deteriorating physiological conditions for the soil microflora, resulting in a decrease in the intensity of the decomposition processes of cellulose-containing organic matter. Thus, the most favorable conditions for the growth of cellulose-degrading microorganisms occur in the upper 0-25-centimeter layer of non-saline soils, especially during the summer. Increased salinization and greater soil profile depth are accompanied by a gradual decline in their abundance, indicating a weakening of the processes of plant residue transformation and the biological cycle of organic matter in saline soils.

Conclusion and Recommendations

The studies conducted showed that the abundance of cellulose-degrading microorganisms decreases systematically as salinity increases and the depth of the soil profile increases. The highest counts were recorded in unsalinated soil in the 0-25 cm layer and amounted to 541,000 in the spring, 641,000 in the summer, and 559,000 in the fall. Under moderate salinity in the same layer, the counts decreased to 230,000, 254,000, and 205,000, respectively that is, by more than half. In the 50-80 cm horizon, the population decreased to 35,000-47,000 under average salinity. Summer proved to be the most favorable season for the development of this group of microorganisms. Overall,

increased salinity and greater soil depth lead to a significant reduction in cellulolytic activity and a slowdown in the processes of organic matter transformation in the soil.

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CHANGES IN THE ACTIVITY OF NITROGEN-FIXING MICROORGANISMS IN IRRIGATED TAKYR IN THE KARAKUL DISTRICT UNDER THE INFLUENCE OF SOIL AND ECOLOGICAL FACTORS

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Annotatsiya. Tadqiqotda sug'oriladigan taqir-o'tloqi tuproqlarda sho'rlanish darajasi, tuproq profili chuqurligi va mavsumiy sharoitlarning azotfiksatsiyalovchi mikroorganizmlar soniga ta'siri o'rganildi. Sho'rlanish kuchayishi va profil chuqurlashishi bilan azotfiksatorlar sonining izchil kamayishi aniqlandi. Eng yuqori ko'rsatkichlar 0-25 sm qatlamda yoz mavsumida qayd etildi. Kuchli sho'rlanishda azotfiksatorlar sonining kamayishi 78,8-86,2 % ni tashkil etib, korrelyatsiya koeffitsientlari $r = 0,989-0,993$ oralig'ida bo'ldi.

Kalit so'zlar. taqir-o'tloqi tuproqlar, sho'rlanish, azotfiksatorlar, tuproq profili, mavsumiy dinamika, korrelyatsiya, biologik faollik.

Аннотация. В работе изучено влияние степени засоления, глубины почвенного профиля и сезонных условий на численность азотфиксирующих микроорганизмов в орошаемых такырно-луговых почвах. Установлено, что с усилением засоления и увеличением глубины численность азотфиксаторов закономерно снижается. Максимальные показатели отмечены в слое 0-25 см в летний период. При сильном засолении снижение численности достигало 78,8–86,2 %, а коэффициенты корреляции составляли $r = 0,989-0,993$.

Ключевые слова. такырно-луговые почвы, засоление, азотфиксирующие микроорганизмы, почвенный профиль, сезонная динамика, корреляция, биологическая активность.

Abstract. The study investigated the effects of salinity level, soil profile depth, and seasonal conditions on the abundance of nitrogen-fixing

microorganisms in irrigated takyr-meadow soils. The number of nitrogen fixers decreased consistently with increasing salinity and soil depth. The highest values were recorded in the 0-25 cm layer during summer. Under strong salinity, the reduction in nitrogen-fixing microorganisms reached 78.8-86.2%, while correlation coefficients ranged from $r = 0.989$ to 0.993 .

Key words. takyr-meadow soils, salinity, nitrogen-fixing microorganisms, soil profile, seasonal dynamics, correlation, biological activity.

Introduction

Nitrogen is one of the key elements in plant mineral nutrition and a crucial factor determining the biological productivity of agroecosystems. Nitrogen-fixing microorganisms play a significant role in maintaining soil nitrogen status; they are capable of binding molecular nitrogen from the atmosphere and converting it into forms available for subsequent use in the soil-plant system. The activity of this functional group of microorganisms depends on a complex set of soil and environmental conditions and can vary significantly under the influence of moisture, temperature, soil pH, organic matter content, nutrient availability, and the degree of salinization [1,2].

The irrigated takyr-meadow soils of the Karakul District form under conditions of a pronounced arid climate, high evaporation rates, and significant anthropogenic pressure associated with the long-term use of these lands for irrigated agriculture. These soils are characterized by heterogeneity in mechanical composition, carbonate content, a relatively low humus content, and a tendency to accumulate easily soluble salts. These properties have a direct impact on the vital activity of the soil microflora and, in particular, on the activity of nitrogen-fixing microorganisms [3,4].

One of the most significant factors limiting the biological activity of irrigated soils in arid zones is salinization. An increase in salt concentration in the soil solution alters osmotic conditions, reduces moisture availability, disrupts ion ratios, and can inhibit the enzymatic and metabolic processes of microorganisms. As a result, the rate of biological nitrogen fixation decreases, which affects plant nitrogen nutrition and the overall level of soil fertility[5,6].

In addition to salinization, seasonal changes in temperature and humidity, soil depth, organic matter content, and the nature of the vegetation cover are of great importance. In the upper soil layers, where the bulk of the roots and plant residues are concentrated, conditions for the development of nitrogen-fixing microflora are generally more favorable. As the soil profile deepens, the amount of available organic substrates decreases, which may be accompanied by a decline in microbiological activity [7,8].

Studying the patterns of change in the activity of nitrogen-fixing microorganisms under conditions of irrigated takyr-meadow soils is of great scientific and practical importance. Such studies make it possible to assess the biological condition of the soil, identify the main factors limiting nitrogen-fixation processes, and determine ways to improve the efficiency of soil-biological processes under arid farming conditions [9,10].

To investigate changes in the activity of nitrogen-fixing microorganisms in irrigated takyr-meadow soils of the Karakul District and to establish the role of key soil-ecological factors in determining their abundance and functional activity.

Materials and Methods

The studies were conducted in the Paykent area of the Karakul District, Bukhara Region, under conditions of irrigated takyr-meadow soils. Plots differing in the degree of salinization unsalinized, slightly salinized, moderately salinized, and highly salinized soils were selected as study sites. When establishing soil profiles, the morphological structure of the profile, the degree of salinization, and irrigation conditions were taken into account. Soil samples for microbiological analysis were collected from the 0-25, 25-50, and 50-80 cm layers during the spring, summer, and fall. The collected samples were placed in sterile containers and transported to the laboratory for further analysis.

The abundance of nitrogen-fixing microorganisms was determined by inoculating a soil suspension onto Ashby's selective medium. To prepare the initial suspension, 10 g of soil was placed in 90 ml of sterile distilled water and shaken thoroughly. Working suspensions were then prepared using a series of tenfold dilutions. One milliliter of suspension was taken from the appropriate dilution and inoculated onto Ashby medium, which is designed to detect free-living nitrogen-fixing microorganisms.

After incubation, the formed colonies were counted, and the abundance of nitrogen-fixing microorganisms was expressed in millions of CFU per 1 g of soil. The obtained data were analyzed according to the degree of salinization, soil horizon depth, and seasons of the year.

Results and Discussion

The results of the experiment show that the abundance of nitrogen-fixing microorganisms in irrigated takyr-meadow soils varies significantly depending on the degree of salinization, the depth of the soil profile, and the season of the year. In all treatment combinations, the highest values were observed in the upper 0-25 cm horizon, whereas as the depth increased to 25-50 and 50-80 cm, the abundance of nitrogen-fixing microorganisms decreased progressively. At the same time, increased salinity was accompanied by a marked decrease in the abundance of this physiological group of microorganisms. In unsalted soil, in the 0-25 cm layer, the abundance of nitrogen-fixing bacteria was 21.8 in the spring, 26.4 in the summer, and 20.6 in the fall. In the 25-50 cm horizon, these values decreased to 13.4, 17.5, and 13.9, respectively, and at a depth of 50–80 cm, to 4.8; 6.2; and 5.0 million CFU/g of soil. This trend indicates a gradual decrease in biological activity with increasing depth (Fig. 1).

In slightly saline soil, the values in the upper horizon were 17.6; 21.1, and 15.8; in the 25-50 cm horizon, 11.2; 13.8, and 9.8; and in the 50-80 cm layer, 3.1; 3.9, and 2.5, respectively. Even at low salinity levels, a noticeable suppression of nitrogen-fixing microflora was observed.

At moderate salinity, the decline was more pronounced. In the 0-25 cm layer, the abundance of nitrogen-fixing microorganisms was 9.2 in the spring, 11.4 in the summer, and 7.8 in the fall. In the 25-50 cm horizon, these values decreased to 5.9, 6.8, and 5.1, and in the 50-80 cm layer to 2.0, 2.8, and 1.6.

The lowest values were observed in heavily saline soil. In the upper horizon, they were 4.5, 6.2, and 3.9; in the 25–50 cm layer, 2.8; 3.4, and 2.3; and in the 50-80 cm horizon, only 0.7; 1.1, and 0.4. This confirms the pronounced inhibitory effect of high salt concentrations on the development of nitrogen-fixing microorganisms.

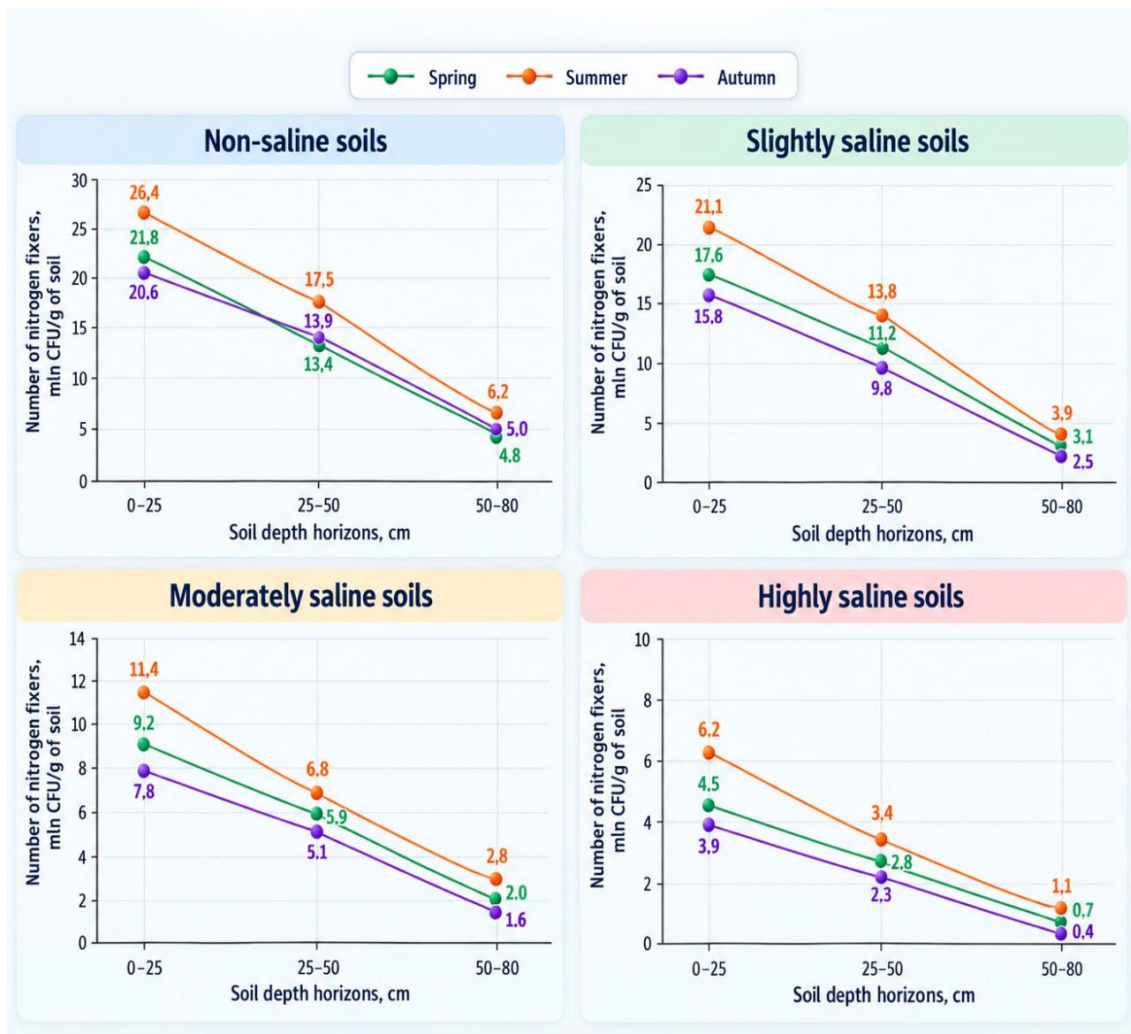


Figure 1. Effect of salinity level on the abundance of nitrogen-fixing microorganisms in irrigated takyr-meadow soils, in millions of CFU/g of soil

Seasonal trends were similar across all treatment groups. The highest values were observed in the summer; in the spring, they were slightly lower, and a decline was noted in the fall. The effect of salinization was most evident in the 0-25 cm layer: in summer, the abundance of nitrogen-fixing microorganisms decreased from 26.4 in unsalinized soil to 21.1 under mild salinization, 11.4 under moderate salinization, and 6.2 under severe salinization. Thus, in the severely saline treatment, the abundance was more than four times lower than in the unsalinated soil. The decrease in the abundance of nitrogen-fixing microflora as salinity increases is associated with a rise in the osmotic pressure of the soil solution, deterioration of the water regime, disruption of the ionic balance, and reduced availability of organic substrates. The decrease in abundance with depth is further due to a reduction in organic matter content and root exudates, as well as poorer aeration of the lower soil horizons.

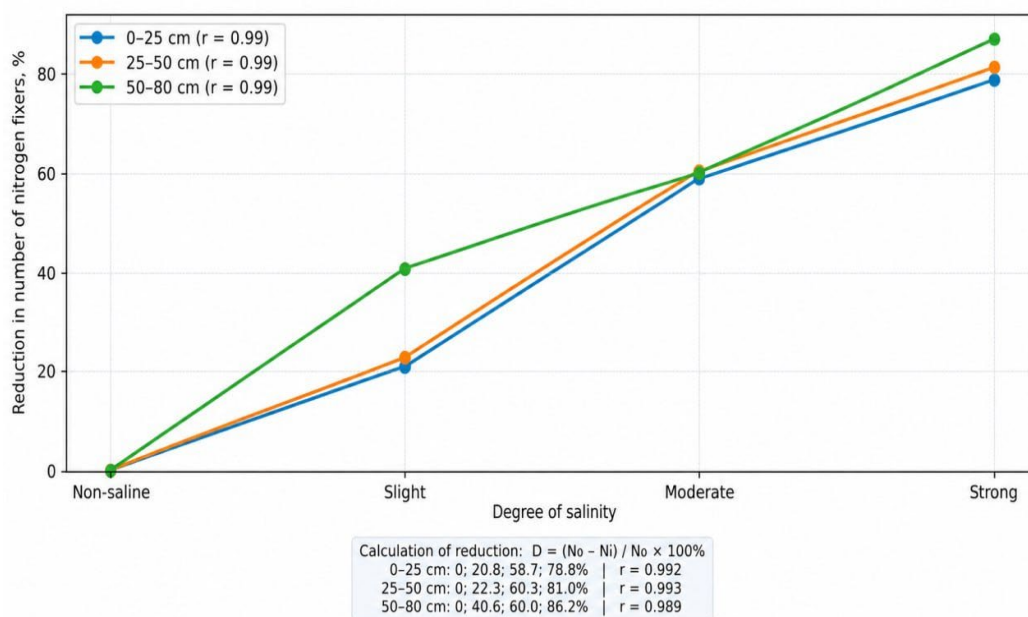


Figure 2. Correlation between salinity levels and the decline in the abundance of nitrogen-fixing microorganisms in various horizons of irrigated takyr-meadow soils

Thus, the degree of salinity and the depth of the soil profile are the main factors determining the abundance of nitrogen-fixing microorganisms in irrigated takyr-meadow soils. The most favorable conditions for their growth occur in the upper, non-saline horizon, whereas severe salinization and increasing profile depth significantly limit their abundance and, consequently, the soil's biological potential. Correlation analysis revealed a close relationship between the degree of salinization and a decrease in the abundance of nitrogen-fixing microorganisms. In all studied horizons, the correlation coefficients were very high, ranging from $r = 0.992$ – 0.993 for the 0–25 and 25–50 cm layers and $r = 0.989$ for the 50–80 cm layer. As the degree of salinization increased, the decline in the abundance of nitrogen-fixing microorganisms intensified: under severe salinization, it reached 78.8% in the 0–25 cm layer, 81.0% in the 25–50 cm layer, and 86.2% in the 50–80 cm horizon (Fig. 2). The data obtained confirm the pronounced effect of salinization on soil biological activity, especially in deeper horizons.

Conclusion and Recommendations

The results of the study showed that increased salinization and greater soil profile depth lead to a decrease in the abundance of nitrogen-fixing microorganisms. The highest values were observed in the 0–25 cm layer during the summer. Under

conditions of severe salinization, the decline in abundance ranged from 78.8% to 86.2%, and high correlation coefficients ($r = 0.989\text{--}0.993$) confirmed a close relationship between salinization and soil biological activity.

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Section 5. ACTUAL PROBLEMS OF MATHEMATICS, PHYSICS AND MECHANICS

UDK: 538.915

CONFIGURATION-DEPENDENT ELECTRONIC STRUCTURE OF OXYGEN-VACANCY- AND TE-RELATED DEFECTS IN α - Al_2O_3 : A FIRST-PRINCIPLES STUDY

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Annotatsiya. DFT (GGA-PBE) yordamida 120 atomli α - Al_2O_3 superyacheykasining elektron tuzilishi nuqsonlarning to'rt oilasi uchun hisoblandi: Frenkel juftiga o'xshash nuqsonlar, kislorod vakansiyalari, tugunlararo Te atrofidagi vakansiyalar va O ning Te bilan almashtirilishi. Toza kristallning zona kengligi 6,17 eV. Kislorod vakansiyalari zona kengligini monoton toraytiradi (4,36 $\rightarrow$ 2,21 eV), Te saqllovchi konfiguratsiyalarda esa javob nomonoton (2,66 dan 1,12 eV gacha), ya'ni uni nuqsonlar soni emas, Te ning lokal koordinatsiyasi belgilaydi. Zona ichidagi holatlar o'tkazuvchi filament hosil bo'lishini o'z-o'zidan isbotlamaydi.

Kalit so'zlar: α - Al_2O_3 , kislorod vakansiyasi, tellur, zichlik funksionali nazariyasi, elektron tuzilish, taqiqlangan zona kengligi, memristor, rezistiv almashinish.

Аннотация. Методом DFT (GGA-PBE) рассчитана электронная структура суперъячейки α - Al_2O_3 из 120 атомов для четырёх семейств дефектов: пар Френкеля, кислородных вакансий, вакансий вокруг межузельного Те и замещения О на Те. Ширина зоны чистого кристалла 6,17 эВ. Кислородные вакансии монотонно сужают зону (4,36 $\rightarrow$ 2,21 эВ), тогда как для конфигураций с Те отклик немонотонен (от 2,66 до 1,12 эВ) и определяется локальной координацией Те, а не числом дефектов. Состояния в запрещённой зоне сами по себе не доказывают образования филамента.

Ключевые слова: α - Al_2O_3 , кислородная вакансия, теллур, теория функционала плотности, электронная структура, ширина запрещённой зоны, мемристор, резистивное переключение.

Abstract. Using DFT (GGA-PBE), we calculate the electronic structure of a 120-atom α - Al_2O_3 supercell for four defect families: Frenkel-pair-like defects, oxygen vacancies, vacancies around an interstitial Te atom, and substitution of O by Te. The pristine gap is 6.17 eV. Oxygen vacancies narrow the gap monotonically (4.36 $\rightarrow$ 2.21 eV), whereas Te-containing configurations show a nonmonotonic response (2.66 to 1.12 eV) controlled by the local Te coordination rather than by the defect count. The in-gap states do not by themselves establish a conducting filament.

Key words: α - Al_2O_3 , oxygen vacancy, tellurium, density functional theory, electronic structure, bandgap, memristor, resistive switching.

Introduction

The memristor, postulated by Chua as the fourth fundamental passive circuit element [1] and identified experimentally in TiO_2 thin-film devices [2], is the basis of resistive random-access memory (RRAM), a leading candidate to complement or replace flash memory and to enable in-memory and neuromorphic computing [3–7]. Its resistance is switched between a high-resistance state (HRS) and a low-resistance state (LRS) by the formation and rupture of a nanoscale conductive filament in the dielectric layer [4,8,9].

Two families of switching mechanisms are distinguished [8,10]. In valence-change memories, the filament is formed by the field-driven drift of oxygen vacancies, as directly observed for Magnéli-phase Ti_4O_7 nanofilaments in TiO_2 [11]; in electrochemical metallization (conductive-bridge) memories, it is formed by cations of an active electrode [10]. In both cases, the energy position and density of defect states in the bandgap determine whether the defect-rich region behaves as an insulator or as a conducting path.

Aluminum oxide (Al_2O_3) is an attractive switching layer because of its wide bandgap (≈ 8.7 eV), relatively high dielectric constant, large band offsets, and thermal

and chemical stability [12–14]. It can be grown by atomic layer deposition (ALD) [15]; ALD Al_2O_3 RRAM cells with a RESET current of about $1\ \mu\text{A}$ [16] and flexible Al_2O_3 memories [17] have been demonstrated, and oxygen vacancies and oxygen-ion drift have been identified as key ingredients of the switching [18,19].

Te-containing electrodes on Al_2O_3 have been used in subquantum conductive-bridge memories [20], and photoemission of $\text{TiTe}/\text{Al}_2\text{O}_3$ cells indicated Te accumulation near the interface, its possible transport into the alumina, and an accompanying formation of oxygen vacancies [19]. For $\text{Al}/\text{Al}_2\text{O}_3/\text{Te}$ structures, a reaction at the $\text{Al}_2\text{O}_3/\text{Te}$ interface under bias, including the possible formation of Al–Te phases, remains a working hypothesis that has not been confirmed experimentally. In any case, Te atoms and oxygen vacancies are expected to coexist in the switching region, yet how they jointly modify the electronic structure of alumina is not known.

In this work, four families of oxygen-vacancy- and Te-related defects in $\alpha\text{-Al}_2\text{O}_3$ (Frenkel-pair-like defects, oxygen vacancies alone, vacancies around an interstitial Te atom, and substitution of O by Te) are compared within one consistent density functional theory (DFT) framework using a 120-atom supercell, to establish whether the number of defects or the local Te–vacancy coordination controls the electronic response.

Literature Review

Bulk electronic structure. $\alpha\text{-Al}_2\text{O}_3$ (corundum, space group $R\bar{3}c$) has an experimental bandgap of $\approx 8.7\ \text{eV}$ [13,21], whereas semilocal DFT gives $6.26\text{--}6.29\ \text{eV}$ [13], $6.50\ \text{eV}$ [22], and $6.045\ \text{eV}$ [23]. This underestimation follows from the derivative discontinuity of the exchange–correlation potential, which semilocal functionals lack [24]. Screened hybrid functionals [25] bring the gap close to experiment [26], but their cost limits the supercell size and the number of configurations, so semilocal calculations remain the practical choice for trends across many defect configurations.

Native point defects. Carrasco *et al.* compared oxygen and aluminum vacancies in the bulk and at the (0001) surface of $\alpha\text{-Al}_2\text{O}_3$ [27]; the neutral oxygen vacancy (F center) introduces a doubly occupied level deep in the gap. Hybrid-functional calculations showed that native defects in $\alpha\text{-Al}_2\text{O}_3$ produce deep levels whose charge state depends on the Fermi-level position [26]. Oxygen-vacancy levels also govern

electron transport through Al_2O_3 [28], and in the amorphous oxide the vacancy is stabilized by a strong local structural rearrangement [29]. The methodology of such calculations is reviewed in Ref. [30].

Oxygen vacancies and conductive paths. Under oxygen-deficient conditions, oxygen vacancies have the lowest formation energy among the intrinsic defects of HfO_2 , TiO_2 , Ta_2O_5 , and Al_2O_3 [31]. In amorphous alumina, vacancy-type oxygen deficiency introduces defect states that may form conductive paths [32], and nanofilamentary switching in binary oxides is attributed to percolating vacancy-rich regions [9,11]. How strongly the gap narrows for a given number of vacancies, and how sensitive it is to their arrangement, is less well established.

Dopants and research gap. Electronegative interstitial dopants (Au, Pt, Pd) in Al_2O_3 lower the formation energy of a neighboring oxygen vacancy by several electronvolts and promote vacancy clustering [18], whereas Te-containing electrodes support conductive-bridge operation with Al_2O_3 [20] and Te can enter the alumina together with oxygen vacancies [19]. However, most first-principles studies consider isolated defects in the dilute limit, and, to our knowledge, none has compared displaced oxygen atoms, oxygen vacancies, interstitial Te, and substitutional Te within one framework at the high local concentrations found in a switching region.

Research Methodology

Calculations were performed with QuantumATK S-2021.06-SP2 [33] (LCAO basis, GGA-PBE [34], PseudoDojo pseudopotentials [35]) for a $2 \times 2 \times 1$ α - Al_2O_3 supercell (R $\bar{3}c$; 48 Al and 72 O atoms; Fig. 1) with a $4 \times 4 \times 1$ k -point mesh, an 85 Ha mesh cutoff, and an SCF tolerance of 10^{-4} Ha. The lattice parameters were fixed at $a = 4.7602$ Å and $c = 12.9933$ Å, and all atoms were relaxed without constraints to forces below 0.05 eV/Å. The calculations were non-spin-polarized; E_g is the indirect Kohn-Sham gap of the supercell, not an optical gap.

Four series were built: (i) Frenkel-pair-like defects, with the O atom nearest the center of the 2nd–6th O layers displaced to the cell boundary ($N_d = 1-8$); (ii) oxygen vacancies only, removed from the center of the second O layer outward ($N_v = 1-7$); (iii) an interstitial Te atom at the cell center with O atoms removed layer by layer ($N_v = 0-$

9); and (iv) substitution of 1–5 central O atoms by Te ($N_{\text{Te}} = 1-5$). These concentrations (up to 12.5% of the O sites) model local defective regions, not the $\text{Al}_2\text{O}_3/\text{Te}$ interface.

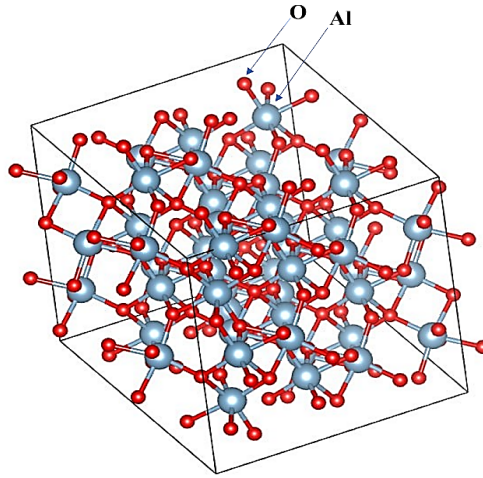


Figure 1. Atomic structure of the $2 \times 2 \times 1$ $\alpha\text{-Al}_2\text{O}_3$ supercell (120 atoms: 48 Al and 72 O) used in all calculations.

Results and Discussion

For pristine $\alpha\text{-Al}_2\text{O}_3$, $E_g = 6.17$ eV and $E_{\text{tot}} = -35667.09$ eV, within 0.3 eV of earlier semilocal results [13,22,23]; only defect-induced changes are discussed below (Table I).

TABLE I. Calculated E_g (eV); pristine: 6.17 eV.

N	(i) N_d	(ii) N_v	(iii) $N_v + \text{Te}$	(iv) N_{Te}
0	6.17	6.17	2.55	—
1	3.85	4.36	2.15	2.55
2	2.47	3.70	1.52	1.89
3	2.04	2.96	2.66	1.81
4	1.09	2.80	2.59	1.18
5	0.87	2.56	2.12	2.43
6	0.89	2.47	1.52	—
7	0.64	2.21	1.51	—
8	2.14	—	1.26	—
9	—	—	1.12	—

In series (i), E_g falls sharply to 0.64 eV at $N_d = 7$ and rises to 2.14 eV at $N_d = 8$ [Fig. 2(a)], as deep levels of displaced O atoms and vacancies form a defect-state manifold; the rebound is tentatively attributed to local relaxation. Because the

composition is conserved, E_{tot} can be compared directly: it increases by ≈ 10 eV per defect [Fig. 2(b)], a static energetic penalty rather than a formation or activation energy.

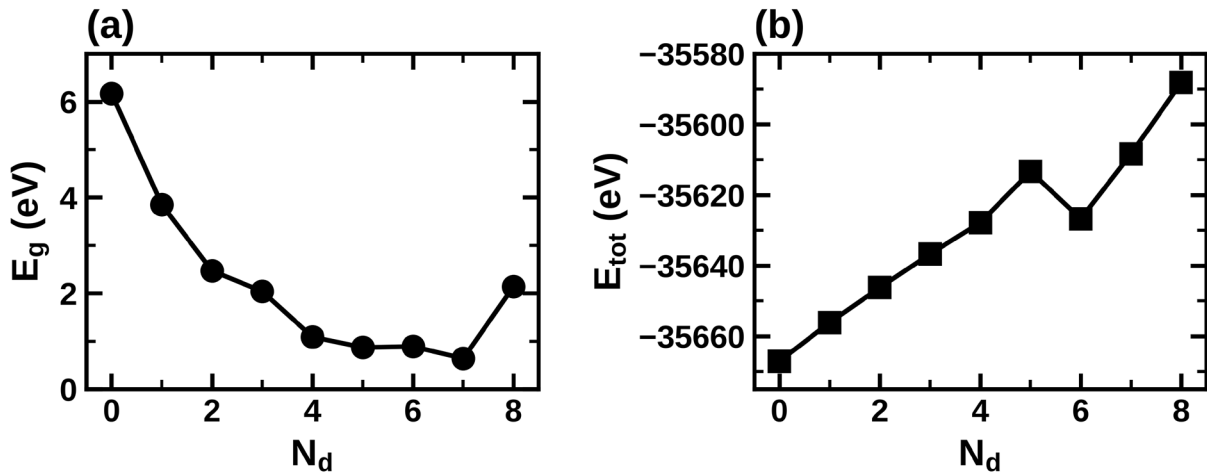


Figure 2. Series (i), Frenkel-pair-like defects: (a) E_g and (b) E_{tot} versus the number of defects N_d .

Oxygen vacancies alone (series ii) narrow E_g monotonically from 4.36 to 2.21 eV [Fig. 3(a)], as vacancy levels merge into a defect-state manifold: no states appear within ± 2 eV of the Fermi level for $N_V = 1$, whereas a dense set is present for $N_V = 7$ [Fig. 4(a,b)]. With interstitial Te (series iii), E_g varies nonmonotonically [Fig. 3(b)]: it falls from 2.55 eV to 1.52 eV at $N_V = 2$, recovers to 2.66 eV at $N_V = 3$, and decreases to 1.12 eV at $N_V = 9$. Thus, Te p -O $2p$ hybridization depends on local symmetry: a favorable arrangement limits defect levels in the gap, whereas further vacancies densify the manifold [Fig. 4(c,d)]. Substitution of O by Te (series iv) lowers E_g to 1.18 eV at $N_{\text{Te}} = 4$ [Fig. 4(e)], with partial recovery to 2.43 eV at $N_{\text{Te}} = 5$. In series (ii)–(iv), E_{tot} mainly tracks the composition and is not a measure of stability. Figure 3(d) compares the four series directly: only series (ii) decreases monotonically, whereas the Te-containing series (iii) and (iv) remain within 1.1–2.7 eV and show pronounced nonmonotonic features. *Limitations.* Comparing compositions requires formation energies, $E_f = E_{\text{defect}} - E_{\text{pure}} + n_{\text{O}}\mu_{\text{O}} - n_{\text{Te}}\mu_{\text{Te}}$ (1), where n_{O} and n_{Te} are the numbers of removed O and added Te atoms and μ is the chemical potential [18,30,36]. Neutral, non-spin-polarized GGA calculations underestimate gaps and omit charge states [28]; one configuration per composition was sampled, and no interface or bias was modeled.

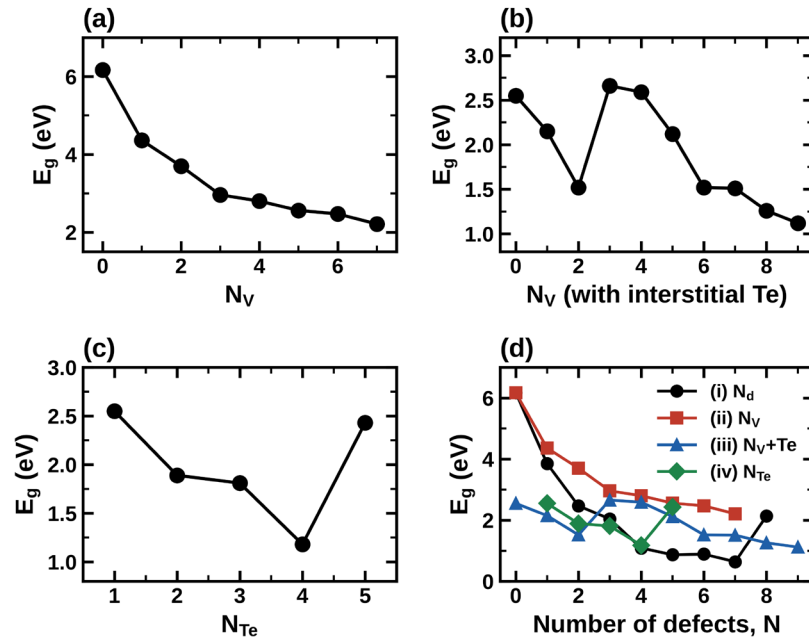


Figure 3. E_g versus the number of defects for (a) oxygen vacancies only (series ii), (b) an interstitial Te atom with oxygen vacancies (series iii), and (c) O → Te substitution (series iv); (d) comparison of all four series.

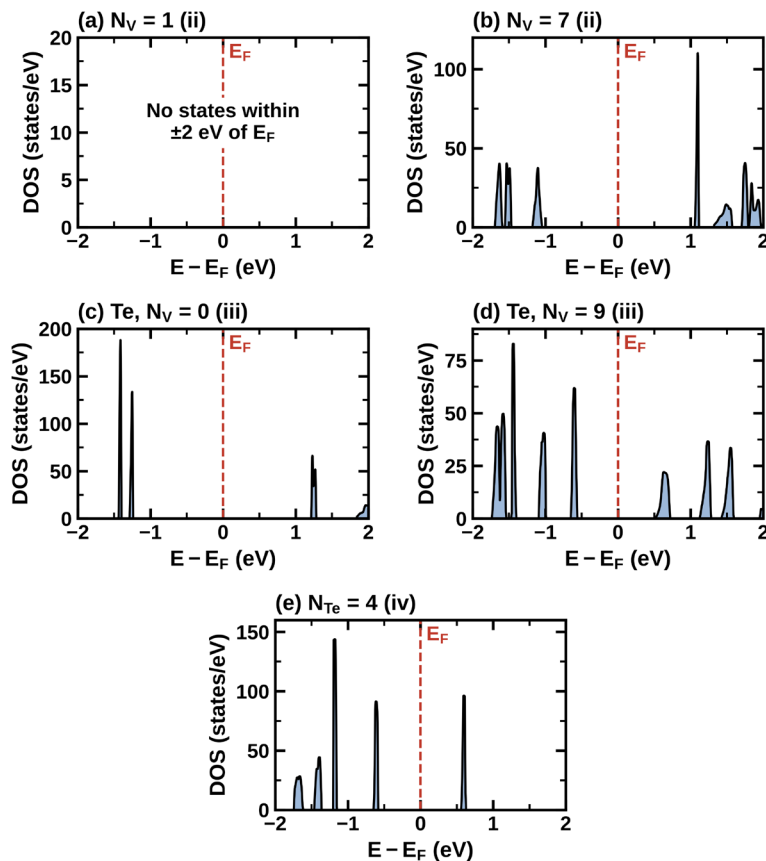


Figure 4. Total density of states near the Fermi level (dashed line, $E_F = 0$) for (a) $N_V = 1$ and (b) $N_V = 7$ (series ii); (c) $N_V = 0$ and (d) $N_V = 9$ with an interstitial Te atom (series iii); (e) $N_{Te} = 4$ (series iv).

Conclusion and Recommendations

Oxygen vacancies narrow the calculated gap of α - Al_2O_3 monotonically ($4.36 \rightarrow 2.21$ eV), Frenkel-pair-like defects narrow it to 0.64 eV, and Te-containing configurations give a nonmonotonic response (2.66 to 1.12 eV) controlled by the local Te–vacancy coordination. Recommendations:

- 1) Several Te–vacancy arrangements and charge states should be sampled per composition when modeling Te incorporation into oxygen-deficient Al_2O_3 .
- 2) Defect stability should be assessed through formation energies (Eq. 1) and spin-polarized calculations with projected analysis of the in-gap states.
- 3) Interface models, migration barriers, and transport calculations are needed to link these states with switching in $\text{Al}/\text{Al}_2\text{O}_3/\text{Te}$ memristors.

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RAPIDLY DECREASING SOLUTIONS OF THE NEGATIVE ORDER MODIFIED KORTEWEG-DE VRIES EQUATION

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Annotatsiya. Ushbu maqolada Zaxarov-Shabat tenglamalar sistemasi uchun sohilish nazariyasidan foydalanib manfiy tartibli modifitsirlangan Korteveg-de Friz tenglamasi yechimini qurish algoritmi keltirilgan. Ishlab chiqilgan algoritm yordamida qaralayotgan masalaning bir solitonli yechimi “tez kamayuvchi” funksiyalar sinfida topilgan.

Kalit so'zlar: modifitsirlangan Korteveg-de Friz tenglamasi, Zaxarov-Shabat tenglamalar sistemasi, xos qiymat, soliton, Yost yechim, spektral maxsuslik.

Аннотация. В работе предложен алгоритм построения решений модифицированного уравнения Кортевега–де Фриза отрицательного порядка, основанный на применении теории обратной задачи рассеяния к системе Захарова–Шабата. В рамках разработанного алгоритма получено явное односолитонное решение рассматриваемой задачи в классе «быстроубывающих» функций.

Ключевые слова: модифицированное уравнение Кортевега-де Фриза, система уравнений Захарова-Шабата, собственное значение, солитон, решение Йоста, спектральная сингулярность

Abstract. In this work, an algorithm for constructing solutions to the negative order modified Korteweg–de Vries equation is proposed, based on the application of the inverse scattering theory to the Zakharov–Shabat system. Within the framework of the developed algorithm, an explicit one-soliton solution of the problem under consideration is obtained in the class of rapidly decaying functions.

Key words: modified Korteweg–de Vries equation, Zakharov–Shabat system, eigenvalue, soliton, Jost solution, spectral singularity.

Introduction

Nonlinear integrable evolution equations play a fundamental role in modern mathematical physics. Classical examples include the Korteweg–de Vries (KdV) equation and the modified Korteweg–de Vries (mKdV) equation, which possess rich analytical structures and admit exact integration. In recent years, increasing attention has been devoted to the study of negative order equations arising within integrable hierarchies. These equations correspond to negative flows of the associated hierarchies and are typically characterized by a more intricate nonlocal structure, distinguishing them from the corresponding positive-order flows.

The study of negative order equations provides a framework for identifying new classes of exact solutions and for elucidating relationships among different integrable models. In particular, negative order equations extend the classical theory of KdV-type equations and provide new perspectives on the structure and dynamics of integrable nonlinear systems. Negative order integrable systems have also been investigated in connection with nonlinear wave phenomena, including shallow-water waves with breaking and peaked profiles, as well as non-smooth wave propagation in nonlinear media. In this context, negative order equations may provide useful mathematical models for wave processes involving strong nonlinear effects, energy localization, and nonstandard dispersive behavior.

The negative order mKdV equation arises as a member of the extended mKdV hierarchy. In contrast to its positive-order counterparts, the negative order mKdV equation generally exhibits a nonlocal structure and may possess qualitatively different dynamical properties. At the same time, it retains the essential features of an integrable system, including a Lax pair representation and the associated spectral formulation. These properties make the negative order mKdV equation an important object for the study of nonlinear wave phenomena and integrable structures.

A central analytical tool for investigating integrable nonlinear evolution equations is the inverse scattering transform (IST). The IST reduces the analysis of a nonlinear evolution equation to an associated linear spectral problem and provides a systematic framework for constructing exact solutions from scattering data. For negative order equations, the spectral problem and the evolution of the scattering data may exhibit additional features arising from the nonlocal nature of the corresponding flow.

Literature Review

The IST method first developed in the fundamental works C.S. Gardner, J.M. Greene, M.D. Kruskal and R.M. Miura [1], V.E. Zakharov and A.B. Shabat [2], M. J. Ablowitz, D.J. Kaup, A.C. Newell and H. Segur [3] for a number of well-known nonlinear models in mathematical physics. The studies [4-5] further illustrate the effectiveness and broad applicability of the inverse scattering method to a wide range of classical nonlinear evolution equations arising in the mathematical modeling of important physical processes and phenomena.

P. J. Olver [6] established a general framework for recursion operators and infinite hierarchies of symmetries of nonlinear evolution equations. Building on this framework, J. M. Verosky [7] introduced negative powers of Olver's recursion operators and established connections between the sine-Gordon and sinh-Gordon equations and potential mKdV equations. Further developments were made in the study of negative order mKdV hierarchies. In the study [8] the negative order mKdV hierarchy was constructed using the inverse recursion operator, and was shown that the sine-Gordon and sinh-Gordon equations arise as members of this negative hierarchy. Subsequently, A.M. Wazwaz [9] developed various negative order KdV and mKdV equations and

investigated their integrability and soliton solutions using recursion operators and related analytical methods. More recent studies [10-11] have also addressed the inverse scattering approach to negative order mKdV equations, including the construction of soliton solutions and the analysis of their Lax representations.

Research Methodology

The inverse scattering transform (IST) associated with the Zakharov–Shabat spectral problem is employed to construct solutions of the negative order mKdV equation in the class of rapidly decreasing functions. The methodology is based on the Lax pair formulation of the equation and consists of the direct scattering problem, the evolution of the scattering data, and the inverse reconstruction of the solution through the Gelfand–Levitan–Marchenko (GLM) equation.

First, the negative order mKdV equation is represented as the compatibility condition of a Lax pair, whose spatial part is a Zakharov–Shabat-type spectral problem. For a given rapidly decreasing real-valued initial function, the corresponding spectral operator is investigated under the assumptions that it has a finite number of simple discrete eigenvalues and no spectral singularities. The direct scattering problem is then solved by introducing the appropriate Jost solutions and determining the associated scattering coefficients, reflection coefficient, discrete eigenvalues, and norming constants. These quantities constitute the scattering data corresponding to the initial potential. Next, the inverse problem is subsequently formulated in terms of the Gelfand–Levitan–Marchenko integral equation. The scattering data at a given time are used to construct the kernel of this equation, whose solution contains sufficient information to recover the potential. The solution of the negative order mKdV equation is then reconstructed from the GLM kernel by means of the corresponding reconstruction formulas. Finally, the developed methodology is applied to a specific rapidly decreasing initial condition for which the scattering data can be calculated explicitly. The resulting GLM equation is solved, and the reconstruction formulas yield an explicit one-soliton solution. This example demonstrates the applicability of the IST–GLM framework to the negative order mKdV equation and provides a basis for its extension to more general negative-order integrable systems.

Results and Discussion

In this work we analyze the following negative order mKdV equation

$$\begin{cases} \eta_x = 2uu_t, \\ u_{xt} + 2\eta u + u = 0, \quad x \in \mathbb{R}, \quad t \geq 0 \end{cases} \quad (1)$$

by the IST method for the Zakharov-Shabat system of equations. We consider the system (1) under the initial condition

$$u(x, 0) = u_0(x), \quad x \in \mathbb{R}, \quad (2)$$

where the initial function $u_0(x)$ is real-valued function having the following properties:

1. $\int_{\mathbb{R}} (1 + |x|)^5 |u_0(x)| dx < \infty$,
2. The operator $L(0) = i \begin{pmatrix} d/dx & -u_0 \\ -u_0 & -d/dx \end{pmatrix}$ possesses a finite number of eigenvalues, precisely, N simple eigenvalues $\xi_1(0), \xi_2(0), \dots, \xi_N(0)$ in the upper half-plane and has no spectral singularities.

Let the functions $u = u(x, t)$ and $\eta = \eta(x, t)$ are sufficiently smooth real-valued functions belonging to the class

$$\mathcal{L} = \left\{ \begin{array}{l} u \in C^2(\mathbb{R} \times (0, \infty)) \cap C(\mathbb{R} \times [0, \infty)), \quad \eta \in C^{1,0}(\mathbb{R} \times (0, \infty)); \\ \lim_{x \rightarrow \pm\infty} \eta(x, t) = 0, \quad t \in \mathbb{R}; \\ \int_{-\infty}^{+\infty} (1 + |x|) (|u(x, t)| + |u_x(x, t)| + |u_t(x, t)| + |u_{xt}(x, t)|) dx < \infty, \quad t \in \mathbb{R} \end{array} \right\}.$$

By performing elementary calculations, it can be shown that the system (1) can be rewritten as the following Lax equation [2] $\dot{L} + [L, M] = 0$, where $L = L(t)$ is defined as $L(t) = i \begin{pmatrix} d/dx & -u \\ -u & -d/dx \end{pmatrix}$, $M = M(\xi, t)$ is matrix having the form

$$M = -\frac{i}{2\xi} \begin{pmatrix} \eta + 1/2 & u_t \\ u_t & -\eta - 1/2 \end{pmatrix}. \text{ Here dot means the derivative with respect to } t.$$

Inverse scattering theory. In this section, we present the basic results of scattering theory for the classical Zakharov–Shabat system, which has been extensively studied in

[2-3]. For simplicity, we omit the time dependence of the functions presented in this section and consider the system in the following form

$$\begin{cases} y_{1x} + i\xi y_1 = u(x)y_2, \\ y_{2x} - i\xi y_2 = -u(x)y_1, \end{cases} \quad x \in \mathbb{R}, \quad (3)$$

where the potential $u(x)$ is real-valued function satisfying condition

$$\int_{-\infty}^{\infty} (1 + |x|) |u(x)| dx < \infty.$$

The system (3) can be rewritten in the form $Ly = \xi y$, where $y = (y_1, y_2)^T$. The system (3) has specific solutions $\phi = \phi(x, \xi)$ and $\psi = \psi(x, \xi)$, usually known as the Jost solutions, which are uniquely determined by the following asymptotics for $\text{Im} \xi = 0$

$$\left. \begin{aligned} \phi(x, \xi) &\sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x} \\ \bar{\phi}(x, \xi) &\sim \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{i\xi x} \end{aligned} \right\}, \quad x \rightarrow -\infty, \quad \left. \begin{aligned} \psi(x, \xi) &\sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi x} \\ \bar{\psi}(x, \xi) &\sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x} \end{aligned} \right\}, \quad x \rightarrow \infty.$$

(Note, that $\bar{\phi}$ is not complex conjugation to ϕ). Since the pair of functions $\{\psi, \bar{\psi}\}$ is linearly independent for real values of ξ , we can write

$$\phi = a(\xi)\bar{\psi} + b(\xi)\psi,$$

where $a(\xi) = W\{\phi, \psi\} = \phi_1\psi_2 - \phi_2\psi_1$, $b = -W\{\phi, \bar{\psi}\}$. The function $a(\xi)$ ($\bar{a}(\xi)$) analytically continues into the upper (lower) half-plane $\text{Im} \xi > 0$ ($\text{Im} \xi < 0$) and has a finite number of simple zeros there. The discrete eigenvalues ξ_k ($\bar{\xi}_k$) of the system (3) coincide with the zeros of the function $a(\xi)$ in the upper (lower) half-plane. Therefore, $\phi_k(x) = B_k\psi_k(x)$, $k = 1, 2, \dots, N$, with $\phi_k(x) = \phi(x, \xi_k)$, $\psi_k(x) = \psi(x, \xi_k)$. The rational-fractional function $a(\xi)$ is reconstructed by its zeros according to the following formula

$$a(\xi) = \exp \left\{ -\frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{\ln(1 + |r^+(z)|^2)}{z - \xi} dz \right\} \prod_{j=1}^N \frac{\xi - \xi_j}{\xi - \bar{\xi}_j}, \quad \text{Im} \xi > 0, \quad r^+(z) = \frac{b(z)}{a(z)}, \quad z \in \mathbb{R},$$

Note that the discrete eigenvalues of the system (3) in the upper and lower half-planes have symmetry relations $\bar{\xi}_k = -\xi_k$ [2]. The set of quantities

$\{r^+(\xi); \xi_k, \text{Im } \xi_k > 0; B_k, k = 1, 2, \dots, N\}$ forms the scattering data for the system (3).

The Jost solution ψ admits the integral representation

$$\psi(x, \xi) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi x} + \int_x^\infty K(x, s) e^{i\xi s} ds, \text{ where } K(x, s) = \begin{pmatrix} K_1(x, s) \\ K_2(x, s) \end{pmatrix} \text{ is a solution of the}$$

following Gelfand-Levitan-Marchenko system of equations on $y > x$

$$\begin{cases} K_2(x, y) + \int_x^\infty K_1(x, s) F(s + y) ds = 0 \\ -K_1(x, y) + F(x + y, t) + \int_x^\infty K_2(x, s) F(s + y) ds = 0, \end{cases} \quad (4)$$

where $F(x) = \frac{1}{2\pi} \int_{-\infty}^\infty \frac{b(\xi)}{a(\xi)} e^{i\xi x} d\xi - i \sum_{j=1}^N c_j e^{i\xi_j x}$, $c_k = \frac{B_k}{\dot{a}(\xi_k)}$, $\dot{a}(\xi_k) = \left. \frac{\partial a(\xi)}{\partial \xi} \right|_{\xi = \xi_k}$. Then

the potential of the system (3) can be reconstructed by the formulas

$$u(x) = -2K_1(x, x), \quad -2K_2(x, x) = \int_x^\infty |u(s)|^2 ds. \quad (5)$$

Based on the inverse scattering theory for the operator $L(t)$ it can be proved the following result

Theorem 1. Suppose that $\{u(x, t), \eta(x, t)\} \in \mathcal{L}$ is solution of the problem (1)-(2) in the class $\mathcal{L}$. Then the scattering data of the operator $L(t)$ satisfy the following evolution equations

$$\begin{aligned} \frac{\partial r^+(\xi, t)}{\partial t} &= \frac{i}{2\xi} r^+(\xi, t), \quad \text{Im } \xi = 0, \\ \frac{d\xi_k}{dt} &= 0, \quad \frac{dB_k(t)}{dt} = \frac{i}{2\xi_k} B_k(t), \quad k = 1, 2, \dots, N. \end{aligned}$$

Based on results of Theorem 1 and the inverse scattering construction, we propose the following algorithm for obtaining rapidly decreasing solution of the problem (1)-(2) in the class $\mathcal{L}$.

Algorithm 1.

1. Find the set of scattering data $\{r_0^+(\xi); \xi_k(0); B_k(0), k=1,2,\dots,N\}$ for a given $u(0)$.

2. Define the function

$$F(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} r_0^+(0,\xi) e^{\frac{i}{2\xi}t + i\xi x} d\xi - i \sum_{k=1}^N c_k(0) e^{\frac{i}{2\xi_k}t + i\xi_k x}.$$

3. Find the function $K(x,y,t)$ by solving the integral equation (4).

4. Recover the function $u(x,t)$ via (5). The function $\eta(x,t)$ is found by

$$\eta(x,t) = \int_{-\infty}^x 2u \cdot u_t dy, \quad \lim_{x \rightarrow \pm\infty} \eta(x,t) = 0, \quad t \in \mathbb{R}. \quad (6)$$

We employ the *Algorithm 1* to construct a single-soliton solution of the problem (1)-(2) in the class of rapidly decreasing functions. For this purpose, we assume that, the system (1) is considered under the initial condition

$$u_0(x) = -\frac{2}{\cosh 2x}. \quad (7)$$

Solving the direct problem for the operator $L(0)$ with potential (7), we have

$$r(x,0) = 0, \quad \xi_1(0) = i, \quad c_1(0) = 2i.$$

Applying the results of Theorem 1, we obtain the following values of scattering data

$$r(x,t) = 0, \quad \xi_1(t) = \xi_1(0) = i, \quad c_1(t) = 2ie^{\frac{1}{2}t}.$$

Solving the system of equations (4) with the obtained results yields

$$K_1(x,y) = \frac{2e^{-x-y+(1/2)t}}{1+e^{-4x+t}}, \quad K_2(x,y) = \frac{-2e^{-3x-y+t}}{1+e^{-4x+t}}. \text{ Then by the formulas (5) and (6), we find}$$

$$u(x,t) = -\frac{2}{\cosh(2x - (1/2)t)}, \quad \eta(x,t) = -\frac{1}{\cosh^2(2x - (1/2)t)}.$$

Conclusion and Recommendations

In this paper, an algorithm based on the IST is developed for constructing rapidly decreasing solutions of the negative order mKdV equation. The proposed algorithm is applied to a specific initial-value problem, yielding an explicit one-soliton solution.

- 1) The inverse scattering framework developed in this paper can be used for further investigation of more general negative order mKdV-type equations with a self-consistent source.
- 2) The developed method can be further applied to the construction and analysis of multi-soliton solutions.

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TRACE FORMULAS FOR THE PERIODIC ABLOWITZ–LADIK SPECTRAL PROBLEM

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Annotatsiya. Ushbu maqolada fokuslovchi reduksiya shartida davriy Ablowitz–Ladik spektral masalasi o‘rganilgan. Monodromiya matritsasi asosida unga mos giperelliptik spektral egri chiziq va yordamchi divizor qurilgan. Davriy potensialni spektral ma’lumotlar bilan bog‘lovchi iz formulalari va fazoviy siljish munosabatlari keltirib chiqarilgan.

Kalit so‘zlar: *Ablowitz–Ladik spektral masalasi, davriy teskari spektral masala, Floquet nazariyasi, giperelliptik spektral egri chiziq, iz formulalari.*

Аннотация. В работе исследована периодическая спектральная задача Абловица–Ладика при фокусирующей редукции. На основе матрицы монодромии построены соответствующая гиперэллиптическая спектральная кривая и вспомогательный дивизор. Выведены формулы следов и соотношения пространственного сдвига, связывающие периодический потенциал со спектральными данными.

Ключевые слова: *спектральная задача Абловица–Ладика, периодическая обратная спектральная задача, теория Флоке, гиперэллиптическая спектральная кривая, формулы следов.*

Abstract. We study the periodic Ablowitz-Ladik spectral problem under the focusing reduction. The associated hyperelliptic spectral curve and auxiliary divisor are constructed from the monodromy matrix. Trace formulas and spatial-shift relations connecting the periodic potential with the spectral data are derived.

Key words: *Ablowitz-Ladik spectral problem, periodic inverse spectral problem, Floquet theory, hyperelliptic spectral curve, trace formulas.*

Introduction

Integrable differential-difference equations constitute an important class of nonlinear systems that combine continuous-time evolution with discrete spatial dynamics. They arise naturally as integrable discretizations of nonlinear evolution equations and provide mathematical models in which essential properties of the corresponding continuous systems, such as Lax representations, conservation laws, and spectral structures, can be preserved. The spectral analysis of such systems is therefore closely related to the theory of integrable nonlinear equations and provides an effective framework for describing their qualitative and algebraic properties.

One of the fundamental spectral problems in the theory of integrable differential-difference equations is the Ablowitz-Ladik spectral problem. It serves as the spectral basis of the Ablowitz-Ladik hierarchy and of a broad class of discrete nonlinear Schrödinger-type systems. In this framework, the dependent variables of the nonlinear equation enter as coefficients of a discrete linear spectral system, while the spectral parameter provides the connection between the nonlinear dynamics and the corresponding spectral characteristics. This makes the Ablowitz-Ladik spectral problem a natural discrete analogue of spectral problems arising in the inverse scattering theory of continuous integrable equations.

The periodic case requires a spectral approach different from that used for rapidly decreasing potentials. For a periodic potential, the basic spectral object is the monodromy matrix obtained by propagating a fundamental matrix solution over one spatial period. Its eigenvalues are the Floquet multipliers, while its trace determines the Floquet discriminant. Consequently, the periodic spectral structure can be described in terms of the monodromy matrix and the corresponding Floquet theory. The algebraic relation satisfied by the Floquet multipliers naturally leads to a two-sheeted spectral curve whose branch points constitute an essential part of the periodic spectral data.

The main purpose of the present work is to establish explicit trace relations between the periodic Ablowitz-Ladik potential and these spectral quantities. By combining the polynomial structure of the monodromy matrix with the branch points and the shifted lifted divisor, we derive relations expressing combinations of

neighboring potential values and the modulus of the potential in terms of spectral data. In this way, the trace formulas provide a direct connection between the coefficients of the periodic Ablowitz-Ladik spectral problem and the algebraic-geometric data associated with its Floquet spectrum.

Literature review

The foundations of integrable differential-difference systems associated with the Ablowitz-Ladik spectral problem were established by M. J. Ablowitz and J. F. Ladik [1,2]. The inverse scattering approach to nonlinear difference equations was further developed in [3], where an integrable nonlinear difference scheme related to the nonlinear Schrödinger equation was investigated. General aspects of soliton theory and the inverse scattering transform were subsequently systematized by Ablowitz and Segur [4]. A detailed spectral and algebro-geometric theory of the Ablowitz-Ladik hierarchy was developed by Gesztesy, Holden, Michor, and Teschl [5-8]. In particular, the recursive construction and zero-curvature formalism of the hierarchy, its hyperelliptic spectral curves, Baker-Akhiezer functions, and algebro-geometric finite-band solutions were investigated in [5,6]. The corresponding algebro-geometric initial-value problem and the inverse spectral reconstruction from divisor data were studied in [7], while conservation laws and the Hamiltonian structure of the Ablowitz-Ladik hierarchy were developed in [8]. A systematic treatment of algebro-geometric methods for discrete integrable systems is also presented in [9].

Periodic spectral methods have also been applied to other integrable differential-difference systems. In particular, periodic solutions of the discrete modified Korteweg-de Vries equation with a self-consistent source were investigated in [10]. These studies provide the spectral and algebro-geometric background for the present work, where the monodromy matrix, Floquet spectrum, hyperelliptic spectral curve, auxiliary divisor, and spatial shifts are used to derive trace formulas for the periodic Ablowitz-Ladik spectral problem.

Research Methodology

The periodic Ablowitz-Ladik spectral problem is investigated using the monodromy matrix and Floquet theory. The associated hyperelliptic spectral curve and

auxiliary divisor are constructed from the normalized monodromy polynomials. Spatial-shift relations and trace formulas are then derived by comparing polynomial coefficients and applying Lagrange interpolation. The investigation is based on the spectral and algebraic analysis of the periodic Ablowitz-Ladik problem under the focusing reduction. A fundamental matrix solution is introduced and propagated over one spatial period to construct the monodromy matrix.

The entries of the fundamental and monodromy matrices are analyzed as Laurent polynomials in the spectral parameter z . The leading and constant coefficients of these normalized polynomials are then identified explicitly in terms of the periodic potential. The focusing reduction additionally yields reciprocal-conjugation relations between the diagonal and off-diagonal polynomial entries. The periodic spectral characteristics are introduced by means of Floquet theory. The eigenvalues of the monodromy matrix are regarded as Floquet multipliers, and its trace defines the Floquet discriminant. Using the determinant of the monodromy matrix, the characteristic equation for the Floquet multipliers is obtained. To introduce the spectral data, the difference between the normalized diagonal entries of the monodromy matrix is considered. The determinant identity then yields an algebraic relation involving this polynomial and the two off-diagonal monodromy polynomials.

The spatial dependence of the divisor is studied by shifting the initial lattice point of the periodic spectral problem. For each lattice index, a shifted monodromy matrix is introduced. Periodicity of the potential implies that two consecutive shifted monodromy matrices are related by a similarity transformation. Consequently, the Floquet discriminant, the determinant of the monodromy matrix, and its branch points remain invariant under spatial shifts. The off-diagonal polynomial and its zeros, however, depend on k , providing the shifted divisor data required for the trace relations.

The trace formulas are derived by comparing coefficients of the normalized spectral polynomials. The leading and constant coefficients of the shifted off-diagonal polynomial yield a product relation between neighboring potential values and the divisor coordinates.

Finally, Lagrange interpolation is applied to reconstruct the required coefficient of this polynomial from its values at the divisor points. Substitution of the resulting expression into the coefficient relation produces the divisor trace formula. Combining this formula with the product relation obtained from the shifted off-diagonal polynomial yields an explicit expression for $|q_k|^2$ in terms of the branch points and the shifted divisor data. Thus, the methodology combines Floquet theory, polynomial identities, spatial shifts, Vieta's relations, and Lagrange interpolation to establish explicit trace relations for the periodic Ablowitz-Ladik spectral problem.

Results and Discussion

We consider the periodic Ablowitz–Ladik spectral problem

$$\Psi_{n+1}(z) = L_n(z)\Psi_n(z), \quad L_n(z) = \begin{pmatrix} z & q_n \\ -q_n^* & z^{-1} \end{pmatrix}, \quad q_{n+N} = q_n,$$

where $z \in \mathbb{C} \setminus \{0\}$.

Let $\Phi_n(z)$ be the fundamental matrix solution normalized by

$$\Phi_0(z) = I_2, \quad \Phi_{n+1}(z) = L_n(z)\Phi_n(z),$$

Then

$$\Phi_n(z) = L_{n-1}(z) \cdots L_0(z) = \begin{pmatrix} A_n(z) & B_n(z) \\ C_n(z) & D_n(z) \end{pmatrix},$$

and the monodromy matrix over one spatial period is

$$\mathcal{M}_N(z) = \Phi_N(z) = L_{N-1}(z) \cdots L_0(z).$$

It is easy to check that $\det L_n(z) = 1 + |q_n|^2$ and we obtain

$$\det \mathcal{M}_N(z) = \prod_{j=0}^{N-1} (1 + |q_j|^2) =: D > 0.$$

Thus, the determinant of the monodromy matrix is independent of the spectral parameter. The entries of Φ_n satisfy

$$A_{n+1} = zA_n + q_nC_n, \quad B_{n+1} = zB_n + q_nD_n,$$

$$C_{n+1} = -q_n^*A_n + z^{-1}C_n, \quad D_{n+1} = -q_n^*B_n + z^{-1}D_n,$$

with $A_0 = D_0 = 1$, $B_0 = C_0 = 0$.

It follows by induction that all four entries are Laurent polynomials in z . Moreover, A_n and D_n contain only powers having the same parity as n , whereas B_n and C_n contain only powers having the parity of $n - 1$.

Introduce $\lambda = z^2$ and define the normalized polynomials

$$\begin{aligned}\mathcal{A}_N(\lambda) &= z^N A_N(z), & \mathcal{B}_{N-1}(\lambda) &= z^{N-1} B_N(z), \\ \mathcal{C}_{N-1}(\lambda) &= z^{N-1} C_N(z), & \mathcal{D}_N(\lambda) &= z^N D_N(z).\end{aligned}$$

Their relevant coefficients are

$$\begin{aligned}\mathcal{A}_N(\lambda) &= \lambda^N - \left(\sum_{j=0}^{N-2} q_{j+1} q_j^*\right) \lambda^{N-1} + \dots - q_{N-1} q_0^* \lambda, \\ \mathcal{B}_{N-1}(\lambda) &= q_0 \lambda^{N-1} + \dots + q_{N-1}, \\ \mathcal{C}_{N-1}(\lambda) &= -q_{N-1}^* \lambda^{N-1} + \dots - q_0^*, \\ \mathcal{D}_N(\lambda) &= -q_0 q_{N-1}^* \lambda^{N-1} + \dots - \left(\sum_{j=0}^{N-2} q_j q_{j+1}^*\right) \lambda + 1.\end{aligned}$$

Furthermore, multiplying the identity $\det \mathcal{M}_N = D$ by z^{2N} gives

$$\mathcal{A}_N(\lambda) \mathcal{D}_N(\lambda) - \lambda \mathcal{B}_{N-1}(\lambda) \mathcal{C}_{N-1}(\lambda) = D \lambda^N. \quad (1)$$

We next introduce the Floquet data. A nontrivial Floquet–Bloch solution satisfies

$$\Psi_{n+N}(z) = \eta(z) \Psi_n(z),$$

and hence $\eta(z)$ is an eigenvalue of $\mathcal{M}_N(z)$. Define the Floquet discriminant by

$$\Delta(z) = \text{tr} \mathcal{M}_N(z).$$

Since $\det \mathcal{M}_N = D$, the Floquet multipliers satisfy

$$\eta^2 - \Delta(z) \eta + D = 0, \quad \eta_{\pm}(z) = \frac{\Delta(z) \pm \sqrt{\Delta^2(z) - 4D}}{2}.$$

Consequently,

$$P_N(\lambda) := z^N \Delta(z) = \mathcal{A}_N(\lambda) + \mathcal{D}_N(\lambda) \quad (2)$$

is a monic polynomial of degree N with constant term one,

$$P_N(\lambda) = \lambda^N + p_1 \lambda^{N-1} + \dots + p_{N-1} \lambda + 1.$$

The Floquet equation determines the two-sheeted algebraic curve

$$y^2 = \Delta^2(z) - 4D.$$

Introducing $Y = z^N y$, we obtain

$$\Gamma: \quad Y^2 = R_{2N}(\lambda), \quad R_{2N}(\lambda) := P_N^2(\lambda) - 4D \lambda^N.$$

Assume that R_{2N} has $2N$ distinct zeros $E_1, \dots, E_{2N}$. Then

$$R_{2N}(\lambda) = \prod_{\ell=1}^{2N} (\lambda - E_{\ell}).$$

To introduce the auxiliary divisor, define $\mathcal{K}_N(\lambda) = \mathcal{A}_N(\lambda) - \mathcal{D}_N(\lambda)$. Combining (1) with (2), we obtain

$$R_{2N}(\lambda) = \mathcal{K}_N^2(\lambda) + 4\lambda \mathcal{B}_{N-1}(\lambda) \mathcal{C}_{N-1}(\lambda). \quad (3)$$

Since, we considering the zeros $\xi_1, \dots, \xi_{N-1}$ of the polynomial $\mathcal{B}_{N-1}$ are distinct and nonzero, then

$$\mathcal{B}_{N-1}(\lambda) = q_0 \prod_{j=1}^{N-1} (\lambda - \xi_j).$$

At $\lambda = \xi_j$, identity (3) gives $\mathcal{K}_N^2(\xi_j) = R_{2N}(\xi_j)$. Hence each zero ξ_j lifts naturally to a point

$$\hat{\xi}_j = (\xi_j, Y_j) \in \Gamma, \quad Y_j = \mathcal{K}_N(\xi_j), \quad j = 1, \dots, N-1.$$

Equivalently, $Y_j = \sigma_j \sqrt{R_{2N}(\xi_j)}$, $\sigma_j \in \{-1, +1\}$.

Thus the direct periodic problem determines the spectral curve and the lifted divisor. We denote the spectral data by $\mathfrak{S} = \{\sigma_j; E_1, \dots, E_{2N}; \xi_1, \dots, \xi_{N-1}\}$.

To describe the spatial dependence of the auxiliary divisor, we consider the shifted spectral problem. Define the shifted monodromy matrix by

$$\mathcal{M}_N^{(k)}(z) = L_{k+N-1}(z) \cdots L_k(z), \quad k \in \mathbb{Z},$$

where $L_{n+N} = L_n$. In particular, $\mathcal{M}_N^{(0)} = \mathcal{M}_N$. Since $\det L_k(z) = 1 + |q_k|^2 > 0$, the matrix $L_k(z)$ is invertible for $z \in \mathbb{C} \setminus \{0\}$, and using $L_{k+N} = L_k$, it is easy to show that the shifted monodromy matrices $\mathcal{M}_N^{(k)}(z)$ satisfy

$$\text{tr} \mathcal{M}_N^{(k)}(z) = \Delta(z), \quad \det \mathcal{M}_N^{(k)}(z) = D.$$

Hence the Floquet polynomial P_N , the polynomial R_{2N} , and the spectral curve $\Gamma: Y^2 = R_{2N}(\lambda)$ are independent of the choice of the initial lattice point. In particular, the set of branch points $\{E_1, \dots, E_{2N}\}$ is invariant under the spatial shift.

Write

$$\mathcal{M}_N^{(k)}(z) = \begin{pmatrix} A_N^{(k)}(z) & B_N^{(k)}(z) \\ C_N^{(k)}(z) & D_N^{(k)}(z) \end{pmatrix}$$

and polynomials satisfy

$$P_N(\lambda) = \mathcal{A}_N^{(k)}(\lambda) + \mathcal{D}_N^{(k)}(\lambda)$$

Define

$$\mathcal{K}_N^{(k)}(\lambda) = \mathcal{A}_N^{(k)}(\lambda) - \mathcal{D}_N^{(k)}(\lambda). \quad (4)$$

Then

$$R_{2N}(\lambda) = \left(\mathcal{K}_N^{(k)}(\lambda) \right)^2 + 4\lambda \mathcal{B}_{N-1}^{(k)}(\lambda) \mathcal{C}_{N-1}^{(k)}(\lambda). \quad (5)$$

The endpoint structure of the shifted problem is obtained from that of the original problem by replacing q_j with q_{j+k} . In particular,

$$\mathcal{B}_{N-1}^{(k)}(\lambda) = q_k \lambda^{N-1} + \dots + q_{k-1}.$$

Assume that $q_k \neq 0$ and that $\mathcal{B}_{N-1}^{(k)}$ has $N-1$ distinct nonzero zeros $\xi_{1,k}, \dots, \xi_{N-1,k}$. Then $\mathcal{B}_{N-1}^{(k)}(\lambda) = q_k \prod_{j=1}^{N-1} (\lambda - \xi_{j,k})$.

At these zeros, (5) gives $\left(\mathcal{K}_N^{(k)}(\xi_{j,k}) \right)^2 = R_{2N}(\xi_{j,k})$. Hence, $Y_{j,k}^2 = R_{2N}(\xi_{j,k})$.

We now derive trace relations connecting the periodic potential with the branch points of the spectral curve and the shifted lifted divisor. We assume that the zeros $\xi_{1,k}, \dots, \xi_{N-1,k}$ of $\mathcal{B}_{N-1}^{(k)}$ are distinct and nonzero. According to the polynomial $P_N(\lambda)$, from the coefficient structure of the monodromy matrix, $p_1 = -\sum_{m=0}^{N-1} q_{m+1} q_m^*$.

On the other hand, using Vieta's formula for the polynomial $R_{2N}(\lambda)$ we have $p_1 = -\frac{1}{2} \sum_{\ell=1}^{2N} E_\ell$. Hence $\sum_{m=0}^{N-1} q_{m+1} q_m^* = \frac{1}{2} \sum_{\ell=1}^{2N} E_\ell$.

For the k -th shifted problem, yields $q_{k-1} = (-1)^{N-1} q_k \prod_{j=1}^{N-1} \xi_{j,k}$.

Define $S_k = (-1)^{N-1} \prod_{j=1}^{N-1} \xi_{j,k}$. Then, $q_{k-1} = S_k q_k$. Next write $\mathcal{K}_N^{(k)}(\lambda) = \lambda^N + \kappa_{1,k} \lambda^{N-1} + \dots - 1$ and comparison of the coefficients of λ^{N-1} with (4) gives

$$q_k q_{k-1}^* = \frac{1}{2} \kappa_{1,k} + \frac{1}{4} \sum_{\ell=1}^{2N} E_\ell. \quad (6)$$

Using Lagrange interpolation we have

$$\mathcal{K}_N^{(k)}(\lambda) = \lambda^N - 1 + \lambda \sum_{j=1}^{N-1} \frac{Y_{j,k} - \xi_{j,k}^N + 1}{\xi_{j,k}} \frac{\prod_{m=1}^{N-1} (\lambda - \xi_{m,k})}{\prod_{m=1, m \neq j}^{N-1} (\xi_{j,k} - \xi_{m,k})}.$$

Taking the coefficient of λ^{N-1} , we find $\kappa_{1,k} = \sum_{j=1}^{N-1} \frac{Y_{j,k} - \xi_{j,k}^N + 1}{\xi_{j,k} \prod_{m=1, m \neq j}^{N-1} (\xi_{j,k} - \xi_{m,k})}$.

Substitution into (6) yields the divisor trace formula

$$q_k q_{k-1}^* = \frac{1}{4} \sum_{\ell=1}^{2N} E_\ell + \frac{1}{2} \sum_{j=1}^{N-1} \frac{Y_{j,k} - \xi_{j,k}^N + 1}{\xi_{j,k} \prod_{m=1, m \neq j}^{N-1} (\xi_{j,k} - \xi_{m,k})}.$$

For convenience, define

$$F_k = \frac{1}{4} \sum_{\ell=1}^{2N} E_\ell + \frac{1}{2} \sum_{j=1}^{N-1} \frac{Y_{j,k} - \xi_{j,k}^N + 1}{\xi_{j,k} \prod_{m=1, m \neq j}^{N-1} (\xi_{j,k} - \xi_{m,k})}.$$

Then $F_k = q_k q_{k-1}^*$, and we obtain $F_k = \overline{S_k} |q_k|^2$, which yields that $|q_k|^2 = \frac{F_k}{\overline{S_k}}$.

Conclusion and Recommendations

The periodic Ablowitz-Ladik spectral problem under the focusing reduction was investigated. Trace formulas relating the periodic potential to the branch points and the lifted auxiliary divisor were derived. The obtained results provide explicit relations between the periodic potential and the corresponding spectral data.

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INVERSE SCATTERING TRANSFORM FOR A NONAUTONOMOUS TODA HIERARCHY FLOW

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Annotatsiya. Ushbu maqolada Toda iyerarxiyasining ikkita kommutativ oqimining vaqtga bog‘liq chiziqli kombinatsiyasi orqali aniqlangan noavtonom sistema tadqiq etilgan. Sochilishning teskari masalasi usuli yordamida Laks tasviri va spektral ma’lumotlarning vaqt bo‘yicha evolyutsiyasi aniqlangan. Qaytish ko‘effitsiyenti va normallash konstantalari uchun aniq formulalar olingan, diskret xos qiymatlarning o‘zgarmasligi ko‘rsatilgan.

Kalit so‘zlar: Toda iyerarxiyasi, sochilishning teskari masalasi, Yakobi operatori, Gelfand–Levitan–Marchenko tenglamasi.

Аннотация. В работе исследована неавтономная система, задаваемая линейной комбинацией двух коммутирующих потоков иерархии Тоды с зависящим от времени коэффициентом. Методом обратной задачи рассеяния получено представление Лакса и определена временная эволюция данных рассеяния. Выведены явные формулы для коэффициента отражения и нормировочных констант, установлена неизменность дискретных собственных значений.

Ключевые слова: иерархия Тоды, обратная задача рассеяния, оператор Якоби, уравнение Гельфанда–Левитана–Марченко.

Abstract. We investigate a nonautonomous system generated by a time-dependent linear combination of two commuting Toda hierarchy flows. Using the inverse scattering transform, we establish the Lax representation and determine the time evolution of the scattering data. Explicit formulas for the reflection coefficient and norming constants are derived, while the discrete eigenvalues remain invariant.

Key words: Toda hierarchy, inverse scattering transform, Jacobi operator, Gel'fand–Levitan–Marchenko equation.

Introduction

Integrable nonlinear evolution equations play an important role in mathematical physics, particularly in the description of nonlinear wave propagation and lattice dynamics. The inverse scattering transform (IST) provides an effective method for solving such equations by relating their nonlinear evolution to the spectral properties of associated linear operators. This approach enables the construction of exact solutions and the analysis of their evolution through scattering data.

An important example of an integrable differential–difference system is the Toda lattice, which describes a one-dimensional chain of particles interacting through an exponential nearest-neighbor potential. Its integrability is closely connected with the discrete Sturm–Liouville (Jacobi) operator. The Toda lattice admits a Lax representation, allowing its nonlinear dynamics to be interpreted as an isospectral deformation of the corresponding Jacobi operator.

For Jacobi operators whose coefficients approach constant background values sufficiently rapidly, the scattering data consist of the reflection coefficient, discrete eigenvalues, and norming constants. The temporal part of the Lax representation determines their evolution. In particular, the discrete eigenvalues remain invariant, whereas the reflection coefficient and norming constants evolve according to relations involving the time-dependent coefficients. The corresponding inverse problem is solved using the discrete Gel'fand–Levitan–Marchenko equation, which reconstructs the Jacobi coefficients from the scattering data.

In this paper, we investigate a nonautonomous Toda flow generated by a time-dependent linear combination of two commuting hierarchy flows. Using the inverse scattering transform, we establish the Lax representation, derive the evolution equations for the scattering data, and reconstruct the solution through the discrete Gel'fand–Levitan–Marchenko equation.

Literature Review

The inverse scattering theory for discrete Sturm–Liouville operators was initiated by Case and Kac [1]. The Toda lattice and its integrable structure were investigated by Toda [2], Manakov [3], and Flaschka [4,5]. The inverse scattering theory for second-

order difference equations was further developed by Guseinov [6]. Ueno and Takasaki [7] introduced the Toda hierarchy, while Teschl [8] provided a systematic treatment of Jacobi operators and integrable nonlinear lattices. The inverse scattering transform for the Toda hierarchy was investigated in [9]. More recently, Zhang and Wang [10] studied variable-coefficient Toda systems. These results provide the theoretical basis for the present investigation of a nonautonomous Toda flow using the inverse scattering method.

Research Methodology

The study uses the spectral theory of Jacobi operators, Lax representation, inverse scattering transform (IST), and the Gel'fand–Levitan–Marchenko equation. These methods are applied to investigate the spectral properties of the nonautonomous Toda hierarchy flow, the time evolution of the scattering data, and the invariance of the discrete eigenvalues.

Results and Discussion

Motivated by these developments, we consider a nonautonomous flow of the Toda hierarchy generated by a time-dependent linear combination of two commuting hierarchy flows. We consider the following system

$$\begin{cases} \dot{a}_n = a_n((a_{n+1}^2 - a_{n-1}^2) + (b_{n+1}^2 - b_n^2) - \gamma(t)(b_{n+1} - b_n)), \\ \dot{b}_n = 2a_n^2(b_n + b_{n+1}) - 2a_{n-1}^2(b_n + b_{n-1}) - 2\gamma(t)(a_n^2 - a_{n-1}^2). \end{cases} \quad (1)$$

where $\{a_n(t)\}_{n \in \mathbb{Z}}$ and $\{b_n(t)\}_{n \in \mathbb{Z}}$ are unknown real-valued functions, and $\gamma(t)$ is a given continuous function.

System of equations (1) is considered under the initial conditions

$$a_n(0) = a_{n0}, \quad b_n(0) = b_{n0}, \quad n \in \mathbb{Z}, \quad (2)$$

where $\{a_{n0}\}_{n=-\infty}^{\infty}$ and $\{b_{n0}\}_{n=-\infty}^{\infty}$ satisfy the following conditions:

1. $a_{n0} > 0$, $b_{n0} \in \mathbb{R}$, $n \in \mathbb{Z}$;
2. $\sum_{n=-\infty}^{\infty} |n| \left(\left| a_{n0} - \frac{1}{2} \right| + |b_{n0}| \right) < \infty$;
3. The operator $L(0)$ has exactly N simple eigenvalues

$\lambda_1(0), \lambda_2(0), \dots, \lambda_N(0)$ lying outside the interval $[-1, 1]$, where

$$(L(t)y)_n = a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1},$$

We assume the existence of a solution of the problem (1)–(2) satisfying the decay and time-regularity conditions below:

$$a_n(t) > 0, \quad b_n(t) \in \mathbb{R}, \quad n \in \mathbb{Z}, \quad (3)$$

and

$$\sum_{n=-\infty}^{\infty} |n| \left(\left| a_n(t) - \frac{1}{2} \right| + |b_n(t)| \right) < \infty. \quad (4)$$

We further assume that $a_n(t)$ and $b_n(t)$ are continuously differentiable with respect to t and that, on every compact time interval I ,

$$\sup_{t \in I} \sum_{n=-\infty}^{\infty} |n| (|\dot{a}_n(t)| + |\dot{b}_n(t)|) < \infty. \quad (5)$$

The main purpose of this paper is to determine the time evolution of the scattering data associated with the Jacobi operator $L(t)$ and to reconstruct the solution of (1) by means of the discrete Gel'fand–Levitan–Marchenko equation.

We briefly recall some well-known results from the scattering theory for the discrete Sturm–Liouville operator

$$(Ly)_n = a_{n-1}y_{n-1} + b_ny_n + a_ny_{n+1} = \lambda y_n, \quad n \in \mathbb{Z}. \quad (6)$$

where $y = \{y_n\}_{n=-\infty}^{\infty}$ is the unknown solution,

$$\lambda = \frac{z+z^{-1}}{2}$$

is the spectral parameter, and

$$a_n > 0, \quad \operatorname{Im} b_n = 0, \quad n \in \mathbb{Z}, \quad \sum_{n=-\infty}^{\infty} |n| \left(\left| a_n - \frac{1}{2} \right| + |b_n| \right) < \infty. \quad (7)$$

In this section, we present some well-known but essential facts concerning the theory of the direct and inverse scattering problems for equation (6), which will be needed in what follows.

Under condition (7), equation (6) possesses Jost solutions with the following asymptotic behavior:

$$\varphi_n(z) = z^n(1 + o(1)), \quad n \rightarrow \infty, \quad |z| = 1, \quad (8)$$

$$\psi_n(z) = z^{-n}(1 + o(1)), \quad n \rightarrow -\infty, \quad |z| = 1. \quad (9)$$

Under condition (7), such solutions exist, are uniquely determined by the asymptotic conditions (8)–(9), and admit analytic continuation into the interior of the unit disk $|z| < 1$. For the function $\varphi_n(z)$, the following representation holds:

$$\varphi_n(z) = \sum_{m=n}^{\infty} K(n, m) z^m,$$

where the coefficient $K(n, m)$ is independent of z and is related to a_n and b_n by the relations

$$a_n = \frac{1}{2} \frac{K(n+1, n+1)}{K(n, n)}, \quad b_n = \frac{1}{2} \left(\frac{K(n, n+1)}{K(n, n)} - \frac{K(n-1, n)}{K(n-1, n-1)} \right).$$

For

$$z \in \mathbb{T} \setminus \{-1, 1\}, \quad \mathbb{T} = \{z \in \mathbb{C} : |z| = 1\},$$

the pairs $\{\varphi_n(z), \varphi_n(z^{-1})\}$ and $\{\psi_n(z), \psi_n(z^{-1})\}$ form fundamental systems of solutions of equation (6). Therefore,

$$\psi_n(z) = \alpha(z) \varphi_n(z^{-1}) + \beta(z) \varphi_n(z).$$

The spectral edge points $z = \pm 1$ are excluded from this representation and, if needed, are treated separately by a limiting argument.

The scattering function is defined by

$$R(z) = \frac{\beta(z)}{\alpha(z)}.$$

It is easy to verify that

$$\alpha(z) = \frac{2}{z - z^{-1}} W\{\psi_n(z), \varphi_n(z)\},$$

where $W\{\cdot, \cdot\}$ denotes the Wronskian.

The function $\alpha(z)$ admits an analytic continuation into the interior of the unit disk $|z| < 1$ and has there a finite number of simple zeros $z_1, z_2, \dots, z_N$. Moreover,

$$\lambda_k = \frac{z_k + z_k^{-1}}{2}, \quad k = 1, 2, \dots, N,$$

are eigenvalues of the operator L . From (13) it follows that

$$\psi_n^k = \beta_k \varphi_n^k, \quad k = 1, 2, \dots, N,$$

where

$$\psi_n^k = \psi_n(z_k), \quad \varphi_n^k = \varphi_n(z_k).$$

If $\lambda_k = \frac{z_k + z_k^{-1}}{2}$ is an eigenvalue of the operator L , then the normalized eigenvector corresponding to this eigenvalue is defined as the vector $\{\xi_n^k\}_{n=-\infty}^{\infty}$, satisfying the condition

$$\sum_{n=-\infty}^{\infty} (\xi_n^k)^2 = 1.$$

Clearly,

$$\xi_n^k = C_k \varphi_n^k, \quad k = 1, 2, \dots, N, \quad n \in \mathbb{Z}.$$

Note that, by virtue of the well-known identity

$$\alpha'(z_k) = -\frac{1}{z_k} \sum_{n=-\infty}^{\infty} \varphi_n(z_k) \psi_n(z_k),$$

we obtain

$$C_k^2 = -\frac{\beta_k}{z_k \alpha'(z_k)}, \quad k = 1, 2, \dots, N.$$

Definition 1.1. The collection

$$\{R(z), z_1, z_2, \dots, z_N, C_1^2, C_2^2, \dots, C_N^2\}$$

is called the *scattering data* for equation (6).

The discrete analogue of the Gelfand–Levitan–Marchenko equation takes form

$$\chi(n, m) + F(n + m) + \sum_{n'=n+1}^{\infty} \chi(n, n') F(n' + m) = 0, \quad m > n, \quad (16)$$

and

$$(K(n, n))^{-2} = 1 + F(2n) + \sum_{n'=n+1}^{\infty} \chi(n, n') F(n' + n),$$

where

$$F(n) = \frac{1}{2\pi i} \oint_{|z|=1} R(z) z^{n-1} dz + \sum_{k=1}^N C_k^2 z_k^n,$$

and

$$\chi(n, m) = \frac{K(n, m)}{K(n, n)}.$$

Now we study the time evolution of the scattering data corresponding to the operator $L(t)$. Let A and $\tilde{A}$ be the difference operators defined by

$$(Ay)_n = a_n y_{n+1} - a_{n-1} y_{n-1},$$

and

$$(\tilde{A}y)_n = c_n y_{n+2} + d_n y_{n+1} - d_{n-1} y_{n-1} - c_{n-2} y_{n-2},$$

where

$$c_n = a_n a_{n+1}, \quad d_n = a_n (b_n + b_{n+1}).$$

The nonautonomous Toda hierarchy flow (1) arises as the compatibility condition of the spectral problem $(Ly)_n = \lambda y_n$ and the auxiliary evolution equation

$$\dot{y}_n = (\tilde{A}y)_n - \gamma(t)(Ay)_n.$$

The following lemma is used to derive the evolution equations for the scattering data.

Lemma 2.1. Let $y_n = y_n(t)$ be a solution of the spectral problem

$$(Ly)_n = \lambda y_n,$$

and define

$$S_n = \dot{y}_n - (\tilde{A}y)_n + \gamma(t)(Ay)_n.$$

Assume that the operator L satisfies the Lax equation

$$\dot{L} = \tilde{A}L - L\tilde{A} + G, \quad G = \gamma(t)(LA - AL).$$

Then S_n is also a solution of the spectral problem, i.e.,

$$(LS)_n = \lambda S_n.$$

Theorem 2.1. *Let $a_n(t)$, $b_n(t)$ satisfy (1) under the assumptions (2)–(5). Let*

$$\lambda = \frac{z+z^{-1}}{2}$$

and let $\alpha(z, t)$, $\beta(z, t)$, z_k , and $C_k^2(t)$, $k = 1, \dots, N$, be the scattering data of the operator $L(t)$. Then, for $z \in \mathbb{T} \setminus \{-1, 1\}$, the continuous scattering coefficients satisfy $\dot{\alpha}(z, t) = 0$,

$$\dot{\beta}(z, t) = \left[\frac{1}{2}(z^2 - z^{-2}) - \gamma(t)(z - z^{-1}) \right] \beta(z, t).$$

and for each discrete eigenvalue $z_k \in (-1, 0) \cup (0, 1)$,

$$\frac{dz_k(t)}{dt} = 0, \quad k = 1, \dots, N,$$

$$\frac{dC_k^2(t)}{dt} = \left[\frac{1}{2}(z_k^2 - z_k^{-2}) - \gamma(t)(z_k - z_k^{-1}) \right] C_k^2(t).$$

Corollary 2.1. *Under the assumptions of Theorem 2.1, the scattering data of the operator $L(t)$ evolve according to*

$$R(z, t) = R(z, 0) \exp \left\{ \frac{1}{2}(z^2 - z^{-2})t - (z - z^{-1}) \int_0^t \gamma(\tau) d\tau \right\},$$

$$z_k(t) = z_k(0) = z_k, \quad k = 1, \dots, N,$$

$$\text{and } C_k^2(t) = C_k^2(0) \exp \left\{ \frac{1}{2}(z_k^2 - z_k^{-2})t - (z_k - z_k^{-1}) \int_0^t \gamma(\tau) d\tau \right\}, \quad k = 1, \dots, N.$$

Remark 2.1. The obtained relations completely specify the time evolution of the scattering data for the operator $L(t)$ and this allows using the inverse scattering method to find the solution of the problem (1)–(4).

Conclusion and Recommendations

The results show that the considered nonautonomous Toda hierarchy flow admits a Lax representation, while the scattering data evolve according to time-

dependent relations and the discrete eigenvalues remain invariant. The obtained results make it possible to reconstruct the system through the inverse scattering method.

The following recommendations are proposed:

1. Extend the results to higher-order Toda hierarchy flows.
2. Construct explicit soliton and multi-soliton solutions.
3. Investigate the obtained results using numerical methods.

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